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An Agent-based Model of Strategic Decision-Making in a City-State System

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Abstract: In the study of historical and strategic decision-making, understanding the complex interactions between military and economic actions within city-state systems is crucial. This paper presents an agent-based model to study the dynamics of city-state systems, focusing on the interaction between military and economic actions in a constrained environment. The model comprises three types of agents: city-states, villages, and battalions. City-states, as the primary decision-makers, can establish villages for food production and recruit battalions for defense and aggression. Simulation data generated through a multi-parameters grid sampling reveal that risk-seeking strategies are more effective in high-cost scenarios, provided the production rate is sufficiently high. The model highlights the significant role of output productivity in shaping strategic preferences, with higher outputs supporting more aggressive expansion and military actions. Conversely, resource limitations compel more conservative strategies focused on survival and resource conservation. The study identifies non-linearities and diminishing returns in strategic investments, emphasizing the need for careful resource allocation.

Keywords: agent-based modelling, strategic behaviour, risk aversion, risk sensitivity, computational history, simulation

Introduction

1.1 In the study of historical and strategic decision-making, understanding the complex interactions between military and economic actions within city-state systems is crucial. These systems, often constrained by limited resources and geographical boundaries, present a unique opportunity to explore how different strategies impact the survival and prosperity of city-states. By examining the dynamic interplay between establishing villages for food production and recruiting battalions for defense and aggression, researchers can gain insights into the decision-making processes that drive expansion or conservation strategies. Agent-based models (ABMs) serve as a powerful tool in this analysis, allowing for the simulation of various scenarios and the observation of emergent behaviors from simple rules governing individual agents. These models have been extensively used to study not only historical phenomena but also contemporary strategic interactions, providing valuable knowledge that bridges the fields of economics, military strategy, and urban planning. There is a long-standing tradition of employing computational methods, particularly agent-based models (ABMs), to gain insights into decision-making processes, resource management, and cooperative behaviors among entities. These models are valuable for studying complex systems, where individual and collective behaviors emerge from the interactions of numerous agents. The flexibility and adaptability of ABMs make them especially suitable for exploring scenarios such as risk-based decision-making, resource competition, and cooperative strategies. Social science has a long-standing tradition of using computational methods, especially agent-based models (ABMs) (Bianchi & Squazzoni 2015; Schelling 1971; Epstein 1999; Retzlaff et al. 2021) These models leverage computational tools and large-scale data to uncover insights into individual and collective human behavior (Mason et al. 2013). In this context, agent-based simulation models are considered to have the capacity to lead to a "generative" approach, referring to the ability to create or generate emergent phenomena from the bottom-up through the interaction of individual agents within a system starting with individual agents and their rules of interaction. (Epstein 1999; Conte & Paolucci 2014; Occa et al. 2022) and to embody an evolutionary perspective (Revay & Cioffi-Revilla 2018; Bertolotti & Roman 2022a). Consequently, ABMs have been applied in various fields, including sociology (Bianchi & Squazzoni 2015), economics (Conte & Paolucci 2014), and archaeology (Romanowska 2021), to study complex systems and validate hypotheses regarding social and historical phenomenaPreiser-Kapeller (2015); Turchin et al. (2018); Klein et al. (2018); Roman & Bertolotti (2023). Agent-based models are powerful tools for studying complex systems due to their ability to simulate emergent behavior, handle heterogeneity, incorporate dynamic adaptation, explore diverse scenarios, and validate hypotheses. Their flexibility and adaptability make them valuable across various fields, providing deep insights into the intricate and often unpredictable nature of complex systems. This interdisciplinary approach is the result of performing predictions and to enhance understanding of phenomena(Chattoe-Brown 2022; Anzola & García-Díaz 2022; Bertolotti et al. 2024).

1.2 Recently, there has been growing interest in using computational methods to understand historical phenomena. Archaeologists have employed agent-based simulation models to validate their hypotheses regarding excavations (Preiser-Kapeller 2015) and study the emergence of trading networks (Ewert & Sunder 2018) and the effects of climate change (Bernigaud et al. 2024)on societal outcomes (Axtell et al. 2002; Graham 2006; Janssen 2009; Romanowska 2021; Ewert & Sunder 2018; Bernigaud et al. 2024). Additionally, agent-based models have been utilized to explore detailed, small-scale conflict scenarios. For example, the study of the ISIS-Kurdish war analyzed the strategic interactions and conflict dynamics at a micro-level, focusing on the specific actions and decisions of individual combatants and units. (Olivia MacmillanScott 2021).

1.3 Since war systems are recognized as complex systems, agent-based models (ABMs) have been extensively utilized to study the strategies and consequences of different combat scenariosXiao-feng (2005); Cornforth (1999); Wang et al. (2012) including real-world armies (Cioppa et al. 2004). These models analyze both small military units and large-scale battles, providing insights into potential alternative outcomes and tactical decision-making processes(Cil & Mala 2010; Scogings & Hawick 2012). Although these models include and analyze tactics to defeat the enemy on the field, this type of competition is tactical rather than strategic, as it omits long-term decisions regarding resources (Walbert et al. 2018). Walbert et al. (2018) presents an ABM simulation based on empirical data to assess how and why nations start a war, considering their network of relationships and wealth accumulation. Furthermore, recent research has explored the benefits of rapidly composable military forces, emphasizing the flexibility of ABMs in simulating heterogeneous and fractionated forces (Timothy R. Gulden 2022).

1.4 In the context of resource competition, city-states can be modeled as agents that perform both military and economic actions because they possess distinct strategic decision-making capabilities, and resource management skills (Cil & Mala 2010). These properties enable city-states to act independently, making strategic choices regarding military engagements and economic policies to optimize their survival and prosperity. City-states generate villages to produce food and battalions for defense and aggression. The competition for resources, such as food and wealth, drives the city-states' strategic decisions, influencing their ability to sustain their population and invest in technological developments (Roman & Bertolotti 2023). These developments enhance the efficiency of food production, wealth generation, and military capabilities(Retzlaff et al. 2021). The strategic attitudes of city-states can be broadly categorized into expansive and conservative, reflecting different risk predispositions based on resource availability(Revay & Cioffi-Revilla 2018; Bertolotti & Roman 2022a).

1.5 The results of this paper reveal counter-intuitive findings regarding the relationship between production costs and strategic behavior. Contrary to expectations, higher production costs lead to a preference for risk-seeking strategies(Mishra 2014). This is explained by the higher marginal advantage of individual units when production and investment are expensive. However, this advantage diminishes if the production rate is too low, as there are insufficient means to achieve adequate production levels. In behavioral terms, city-states adopt risk-seeking strategies when investment costs are high, but only if their production rate is sufficient to support such investments(Arend Hintze 2015). This finding has significant implications for understanding the strategic behavior of city-states and the factors that drive their decision-making processes (Bertolotti & Roman 2022b; Ogibayashi 2022). This observation aligns with broader discussions on risk-averse behaviors in various social contexts, highlighting the role of ABMs in elucidating these dynamics (Smaldino 2023; O'Sullivan 2023; Namid Stillman & Gleiser 2023).
Starting with the Methods section, it details the computational method of ABM, explaining its characteristics such as agent autonomy, heterogeneity, interaction, and emergence, which are crucial for understanding complex systems. The model description follows, presenting a two-dimensional, topologically explicit system of city-states C_i, villages V_i, and battalions B_i. It outlines the state variables and strategic and tactical parameters influencing city-state C_i decisions on resource allocation and military actions. Then, the Results section

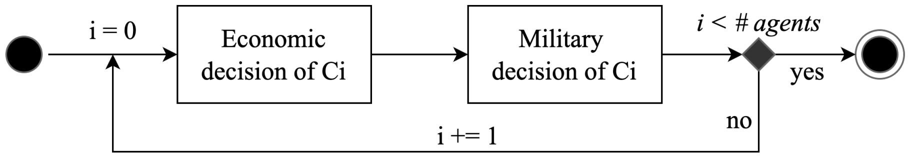


Figure 1: Model scheduling for a single time-step for a city-state C_i

reveals the findings on the relationship between production costs and strategic behavior, emphasizing the role of output productivity in shaping strategies. Finally, the Discussion and Conclusions section interprets these results, highlighting the significant impact of economic productivity on city-state strategies and the importance of balancing resource allocation to achieve optimal outcomes.

Methods

2.1 Agent-Based Modeling is a computational method for simulating interactions of agents within a system to assess their effects on the system as a whole (Bonabeau 2002). It is characterized by various aspects, including agents (Walbert et al. 2018), which are individual entities with distinct behaviors and characteristics, and autonomy (Wang et al. 2012), as agents operate independently, making their own decisions based on a set of rules or algorithms. There is also heterogeneity among the agents (Timothy R. Gulden 2022), who differ from one another in various ways such as their attributes, behaviors, and decision-making processes. Interaction plays a crucial role (Scogings & Hawick 2012), both between the agents themselves and with their environment, leading to emergent behavior at the system level. The environment, the context or space within which agents operate, can be physical, social, or virtual. Finally, emergence is a key feature, where complex system behaviors and patterns arise from the simple interactions of individual agents, often resulting in unexpected outcomes. These characteristics make agent-based modeling a powerful tool for understanding complex systems and exploring the dynamics of agent interactions.

Model description

2.2 The ABM presented here depicts the interactions between various city-states in a constrained space, considering different factors. The model portrays a two-dimensional and topologically explicit system, i.e., the fact that agents have a certain position in a space, illustrating the spatial positioning of city-states competing for limited space, with no possibility for the establishment of new city-states.
Within this system, city-states have the capacity to sustain food production through villages, directly influencing population growth and positively impacting various economic aspects. Furthermore, the model includes the potential for military conflicts among city-states. At its current stage, the model does not encompass other forms of interactions. The primary goal of the model is to analyze the survival patterns of different types of city-states in diverse environmental settings, aiming to derive comprehensive insights into competition within a confined environment characterized by scarce resources.

2.3 The model is designed to study the potential paths available to individual city-states in terms of their economic development, military strategies, and resource management to secure their survival and achieve prosperity over a specified period. It incorporates three distinct types of agents: city-states C_i, villages V_i and battalions B_i. City-states are the primary decision-makers, undertaking a wide range of actions to drive their development. The scheduling of these actions for a single time step is illustrated in Figure 1.
Each city-state C_i is defined by the states depicted in Table 1. These states can undergo both endogenous changes and exogenous changes, related to the interaction processes between city-states. The population stock $p_i(t)$, standing for the number of tax-generating citizens in the city-state who are also available for enrolment is an example of endogenous change. It increases when a certain amount of food is available in the city-state to cover the food needs of the citizens and soldiers stationed to defend the city. The gold $g_i(t)$ of a City-state C_i is related to the population by a positive dependence on the fact that this increases with the collection of taxes, which is directly proportional to the population level in the city.
The states variables $w_i(t)$, $ct_i(t)$, $mt_i(t)$ and $cd_i(t)$, which respectively represent the general wealth level of the

Name	Description
$g_i(t)$	Gold in the city
$f_i(t)$	Food in the city
$mfc_i(t)$	Max food capacity
$p_i(t)$	Population of the city
$w_i(t)$	Wealth of the the city
$ct_i(t)$	Civil technology of the city
$mt_i(t)$	Military technology of the city
$cd_i(t)$	Defence of the city

Table 1: List and description of state variables for a city-state C_i

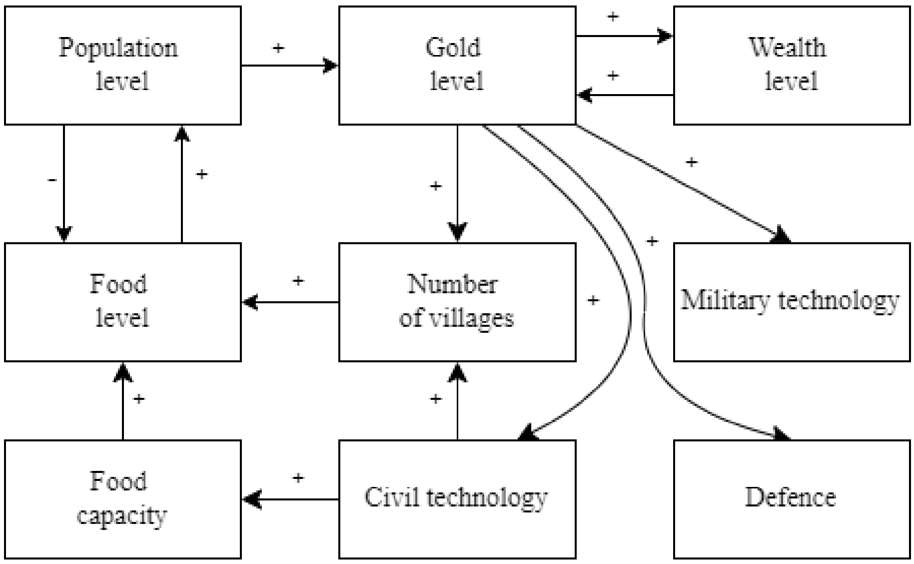


Figure 2: Graph of functional dependencies depicting the economical dynamics of a city-state C_i

population influencing the amount of gold the $p_i(t)$ produces, the technological level in the civil field directly responsible for $mfc_i(t)$ and the costs of founding a new V_i, the technological level in the military field improving the fighting statistics of battalions and the city's defences which provide enhancements to the statistics of garrison soldiers during the city's defense phase, only undergo positive increments whenever the city-state decides to embark on a development phase compatible with the available resources. Figure 2 shows the economic dynamics of a city-state agent, highlighting the functional dependencies that each different state variables have on each other.

2.4 City-states C_i operate as decision-making entities, constantly making choices on how to allocate resources. At each time-step, they must decide whether to invest resources or create villages V_i or battalions B_i, and how to utilize these battalions B_i. The economic phase of a city-state's decision-making process is composed of two stages. Initially, the C_i gathers gold $g_i(t)$ and food $f_i(t)$, influenced by its gold-rate, population $p_i(t)$ values, and village production. Subsequently, the city-state C_i deliberates whether to enhance wealth $W_i(t)$, technology $(mt_i(t), ct_i(t))$, or defense $cd_i(t)$, construct a battalion B_i, or establish new villages V_i. On the other hand, the military phase involves determining the battalion's course of action: the city-state can launch missions to directly assault enemy city-states C_i or their villages V_i, or deploy battalions B_i to safeguard a village V_i and shield it from potential enemy incursions. The mission is the method found to make B_i move together and rationally, in a way that is consistent with the city-state's strategy. Each decision is influenced by specific internal or external conditions and a set of behavioral parameters. These behavioral parameters can be further categorized into strategic parameters (Table 2) and tactical parameters (Table 3). Tactical parameters are specific details and immediate conditions that influence the conduct of military operations on the battlefield. In this model, they are the target decision coefficients. In contrast, strategic parameters are concerned with long-term planning and broader considerations that affect the overall conduct of a war or campaign. These parameters are focused on achieving large-scale objectives and can influence the overall outcome of the sim-

Name	Description	Allowed Values
pv_i	Preference to found a village	$pv_i \in [0,1]$
pct_i	Preference to invest in civil technology	$pct_i \in [0,1]$
pmt_i	Preference to invest in military technology	$pmt_i \in [0,1]$
pw_i	Preference to invest in wealth	$pw_i \in [0,1]$
pd_i	Preference to invest in defences	$pd_i \in [0,1]$
pb_i	Preference to recruit a battalion	$pb_i \in [0,1]$
pp_i	Preference to send protecting troops	$pp_i \in [0,1]$
pm_i	Preference to organize a mission	$pm_i \in [0,1]$
pva_i	Preference to attack a village	$pva_i \in [0,1]$
pca_i	Preference to attack a city-state	$pca_i \in [0,1]$

Table 2: List and description of strategic parameters for a city-state C_i

Name	Description	Allowed Values
α_1	Coefficient of target decision regarding enemy's defence	$\alpha_1 \in [-1,1]$
α_2	Coefficient of target decision regarding enemy's number of battalions	$\alpha_2 \in [-1,1]$
α_2	Coefficient of target decision regarding enemy's distance	$\alpha_3 \in [-1,1]$
α_4	Coefficient of target decision regarding enemy's military technology level	$\alpha_4 \in [-1,1]$
α_5	Coefficient of target decision regarding enemy's gold	$\alpha_5 \in [-1,1]$
α_6	Coefficient of target decision regarding enemy's food	$\alpha_6 \in [-1,1]$
α_7	Coefficient of target decision regarding enemy's population	$\alpha_7 \in [-1,1]$

Table 3: List and description of tactical preference parameters for a city-state C_i

ulation. In this model, these are the preferences of ordinary actions. The strategic parameters can take on a value $x_i \in R : x_i \in [0,1] \wedge \sum x_i = 1$, with the exception of pva_i and pca_i, which can take on a value $y_i \in R : y_i \in [0,1] \wedge \sum y_i = 1$. This difference arises from the fact that pva_i and pca_i are related to the city's inclination to directly attack enemy city-states or their villages. These values are subordinate to the value of pm_i, which represents the city-state's inclination for organizing offensive missions.
Once the mission has been organized, the city-state must decide which type of target to direct its attack toward. These parameters determine the strategy each City-state C_i chooses to adopt for resource management. For example, if $pv_i = 0.2$, it means that the likelihood for a city-state C_i to construct a new village during the economic phase of the decision-making process, and only if the option is available, is $P(v) \propto 0.2$.

2.5 The tactical parameters can take on a value $z_i \in R : z_i \in [-1,1]$ and are utilized to determine which enemy to attack after the decision to attack has been made. Each parameter serves as a multiplier for specific characteristics of the enemy C_i it interacts with. The sum of these values forms a final score, and C_i will select the enemy with the highest score to attack. Every value is compared with the total amount present on the map. For instance, α_2 multiplies the number of B_i each C_i possesses by the total number of B_i on the map. This helps determine the target's "danger level". The tactical parameters $z_i \in R : z_i \in [-1,1]$ were thought to explore which characteristics of the target city-states were given the most consideration. Each C_i will have its own unique preferences, assigning varying levels of positive or negative importance to different aspects. These parameters play a important role in the model. Given that attacking is the sole mode of interaction in the model, and that each city-state C_i has its own unique set of parameters, behavioral parameters govern the target selection decision and influence also how the economic outputs of two city-state agents are stressed.
Finally, there are some environmental parameters of interest (Table 4), such as the initial number of city-states C_i the rate of production of the two resources (respectively pgp for the gold and bvp for the food), and the cost of production of a battalion bsc. Villages V_i play an essential role as food producers, supplying the necessary resources to sustain the population $p_i(t)$ of the city-states C_i and its battalions B_i. Villages V_i have a basic level of food production, bvp, that will be multiplied by a fertility factor, characteristic of the specific patch of the village V_i. Villages V_i can be defended by the city-state's battalions B_i or destroyed by enemies battalions B_i.

Name	Description	Allowed Values
N	Number of starting city-states	N [5, 20)
bsc	Base battalion recruitment cost	bsc [20000, 2000000]
pgp	Person gold production	pgp [1, 1000]
bsp	Base village food production	bvp [1 , 1000]

Table 4: List and description of environmental parameters

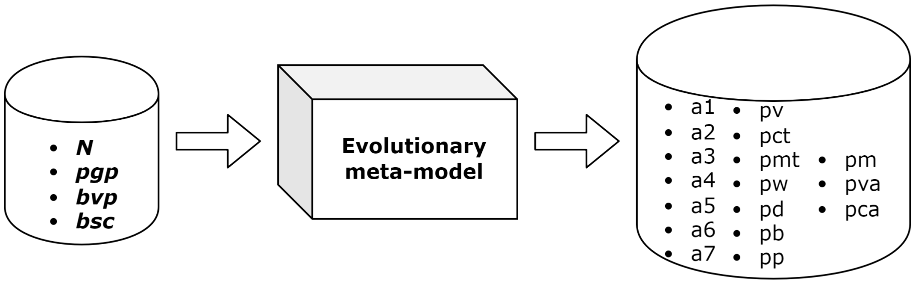


Figure 3: Graph of functional dependencies depicting the economical dynamics of a city-state C_i

Battalions B_i are enlisted by the city-states C_i to protect against external threats or to launch military offensives against other city-states C_i and their villages V_i. Battalions B_i have a level, which is the military technology level, $mt_(t)$ of its owner, that determine the damage inflicted and received by enemies battalions B_i. Once the health of the battalion reaches 0 the battalion disappears.
Each type of agent contributes distinctively to the simulation, influencing the overall dynamics of resource management, economic growth, and military strategy.

Experimental setups

2.6 The model outlined in the preceding was implemented in NetLogo 6.3.0. The choice was influenced by its user-friendly interface, the possibility to easily share and communicate the model, and the fact that only a limited number of agents are simulated, thus obviating the need for high-performance computing. The experimentation phase employed NetLogo's BehaviorSpace, which permits to perform a grid sampling exploration of the parameter space. The model was run 1250000 times, and the results collected with the different input. This high number of repetitions was pivotal in ensuring the statistical robustness of our results and allowed us to comprehensively explore the ramifications of various input variables on the intricate interactions among city-states, villages, and battalions through in-depth simulation data analysis.

2.7 In a black-box perspective , Figure3, the model exploration consists in sampling eight key inputs, with each input variable varied across a specified large range to cover both extreme and moderate values. The decision to use a random grid sampling system was guided by the fact that, without knowing what result to expect beforehand, it was considered the best way to examine as many combinations as possible and discover interesting patterns within the model. Each variable was collected from a uniform random distribution, which was employed because the exploration did not cover multiple order of magnitude, and so a log-uniform distribution was not needed. For each simulation run, data was collected on key outcome variables for the surviving city-states. This data collection process enabled statistical analyses that generate information regarding how different environmental parameters influenced the overall dynamics of the system. These experiments allowed us to observe how different scenarios impact city-states' preferences, resource management, and overall economic and military dynamics. The data was treated and analyzed using Python 3.12.2 in a Jupyter Notebook. Specifically, using libraries: numpy 1.26.4, pandas 2.2.0, matplotlib 3.9.0, plotly 5.22.0 and seaborn 0.13.2.

Results

3.1 Figure 4 displays the distribution of the share of C_i that survived in each simulation run. The x-axis represents the share of city-states that survived, and the y-axis represents the number of simulations where a given share of C_i survived. The distribution is right-skewed, with an higher frequency of simulations where only few city-states survived. There is a visible peak at 1.0, with around 0.025 relative frequency, indicating that in some simulations, all C_i survived. This suggests certain conditions or scenarios where complete survival was achieved. The peak of values close to the first bin suggests an higher likelihood of scenarios where just one C_i survived, possibly indicating extremely aggressive strategies by city-state C_i. The histogram provides a comprehensive view of the survival rates of C_i, showing a predominant trend towards lower survival rates, implying total domination by a C_i , with a descendent trend for higher survival rate share.

The study of environmental parameters can provide deeper insights into this distribution. Analyzing how the

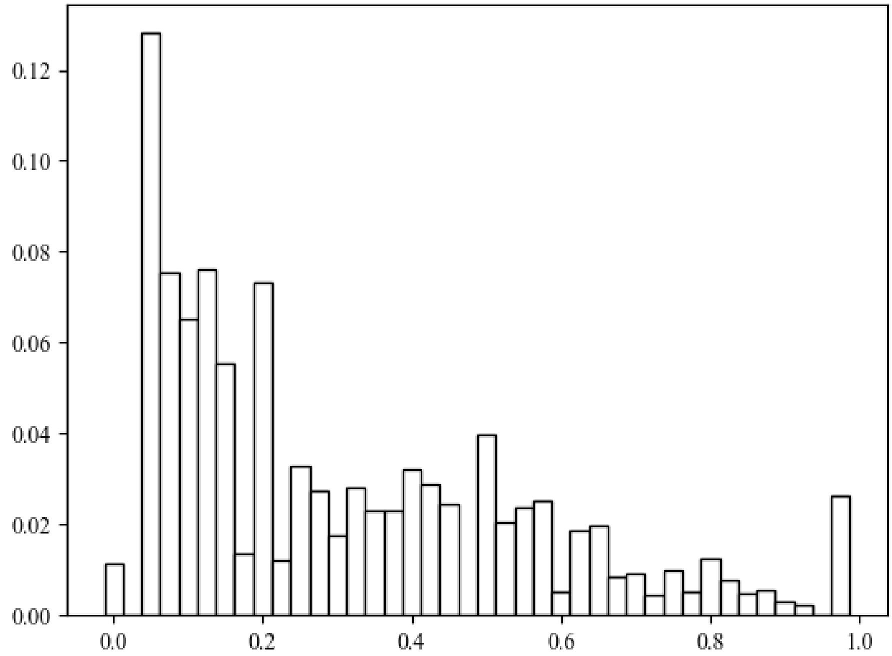
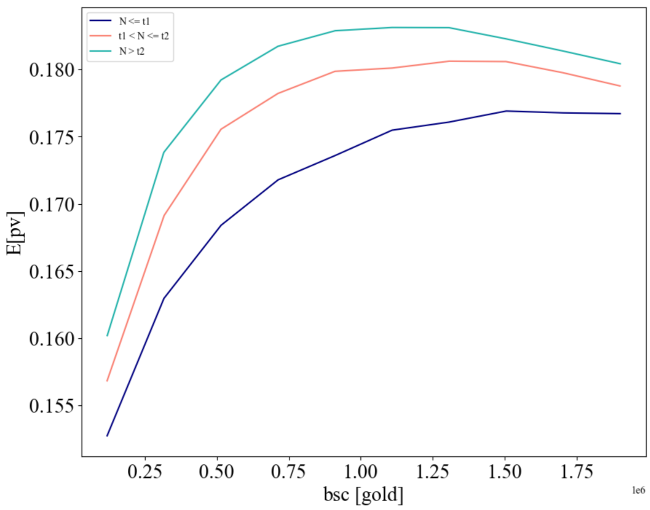


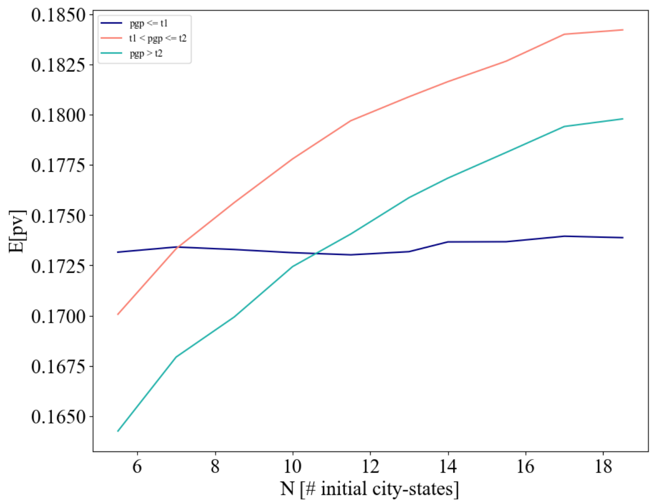
Figure 4: Distribution of the share of City-states C_i survived until the end of the simulation with respect to N

set of preferences of city-states C_i changes in relation to different values of environmental parameters will give us a better comprehension of which strategic strategy city-states tend to adopt in different scenarios. The goal is to determine the best strategy (i.e. expansive or conservative) and summarize them into a matrix. The following analysis, depicted in Figure 5a, Figure 5b, Figure 5c, and Figure 5, involves plotting a behavioral output on the y-axis, while observing the co-effect of two different inputs: one on the x-axis and the other used to divide the data into three clusters by tertiles, which boundaries are respectively called t_1 and t_2 for each variable. On the x-axis, a significant environmental parameter is represented in each graph in its range of simulated values. This approach allows us to study the behaviour of the function on the y-axis (i.e. the most relevant city-states preferences) within the two most extreme values of the environmental parameter. The clustering plotting gives more detail and allows a third level analysis of the graph. These graphs below represent on the y-axis the average values of pv and pm (the preference for founding a village and the preference for organizing a military mission respectively), indicated respectively as $E[pv]$ and $E[pm]$. The choice of the environmental parameters is strategic. These parameters can be divided on the base of their effect on the city-states. On the one hand, some parameters have an internal effect (i.e. pgp or bvp) reflecting the production capability of the city-state, on the other hand, some depict the external effect reflecting the global costs or the resources scarcity of the environment (i.e. N or bsc).

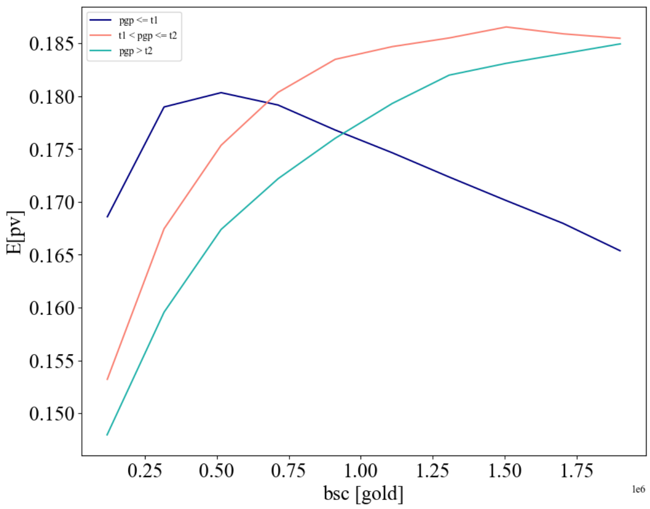
3.2 Figure 5a depicts the relationship between bsc and pv, clustered by N. For all clusters of N, $E[pv]$ initially increases rapidly with bsc. Each line exhibits a peak followed by a subsequent decline, indicating a non-monotonic behavior. The peaks occur at different bsc values for each cluster, with the highest N cluster peaking at the highest bsc value, followed by the medium N cluster and the lowest N cluster. This indicates that the higher the number of N, the greater the preference for founding villages is. Larger C_i seem to sustain a higher preference for a higher cost longer than smaller C_i. This can be connected to the varying success of different risk-related attitudes (Arend Hintze 2015). Notably, as bsc increases, a more expansive and risk-prone strategy emerges, aiming to seize as much territory as possible by founding villages until the area is saturated. Economic expansion is the key strategy city-states adopt to sustain their economy in an environment with such high external costs. Since they have an increase in food production they can sustain more population which inevitably leads to a bigger gold production and in the end the capability to spend this gold on other activities. The non-monotonic behavior of $E[pv]$ suggests that after a certain point, increasing the pv_i leads to a decrease in relative value. This could be due to diminishing returns or increased vulnerability. Since each C_i can only perform one action per turn, it exposes itself to the risk of enemy offensives targeting its villages. This occurs because the city-state would be less protected due to its lower pb_i (preference of recruiting battalions) in favor of pv_i (preference of founding new villages). Extreme values of pv_i might bust the economy of the city-state C_i in an initial phase but put it in a very uncomfortable position once the other city-states C_i reach a point where they start to attack



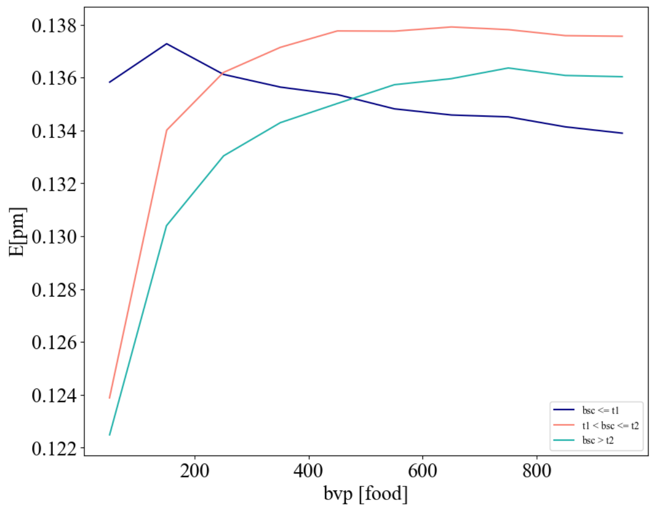
(a) Line plot of $E[pv]$ of C_i related to bsc clustered by N



(b) Line plot of $E[pv]$ of C_i related to N clustered by pgp



(c) Line plot of $E[pv]$ of C_i related to bsc clustered by pgp



(d) Line plot of $E[pm]$ of C_i related to bvp clustered by bsc

Figure 5: Line plots of $E[pv]$ and $E[pm]$ related to different clusters and x-axis

their neighbors. In this scenario, the excessive pv_i undermines the city-state's capability to recruit an effective amount of battalions B_i to protect itself or to counterattack.

Figure 5b illustrates the relationship between the number of initial city-states N and pv, with data clustered by the amount of pgp. For $pgp > t1$, pv increases with N. However, the rate of increase and the starting points differ among the clusters. For $pgp < t1$, the trend is almost flat, indicating a negligible relationship between N and pv_i.
A higher N on the map implies greater resource scarcity, which in turn increases the value of these resources. Consequently, the propensity for expansion grows as N increases. This trend is evident in the clusters where $pgp > t1$. Conversely, when the city's ability to generate resources (pgp) remains low, the tendency to expand diminishes, reflecting the lack of necessary resources to support such expansion.
The graph indicates that economic strength, as represented by pgp, significantly influences the relationship between N and pv_i. Higher economic productivity buffers the impact of increased N, sustaining the expansion preference longer. The high number of initial city-states N creates a very competitive environment where only a rapid economic expansion seems to be the effective survival strategy. On the other hand, if the internal production is not big enough to sustain that kind of strategy city-states C_i tend to adopt a more balanced approach avoiding excessive exposition on one particular preference. For $pgp > t1$, there is a clear, non-linear growth relationship between N and pv_i more like a logarithmic scale. This relationship saturates after a certain level,

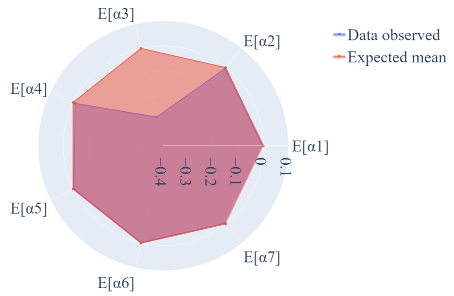
suggesting diminishing returns on expansion investment.

Figure 5c depicts the relationship between bsc and pv_i, clustered by pgp. For $pgp > t1$, there is a non-linear, increasing relationship between bsc and pv_i. This relationship peaks and then saturates, indicating diminishing returns on investment in soldier recruitment after a certain point. For $pgp < t1$, the saturation occurs much earlier, and pv_i values start decreasing significantly even at lower bsc levels. Economic strength, represented by pgp, plays a crucial role in buffering the impact of bsc. Higher pgp allows for a more extended increase inpv_i as bsc rises, reflecting a sustained capacity for expansion. In contrast, lower pgp leads to early saturation and decline, showing that C_i with lower economic productivity cannot maintain high costs for long.
The graph highlights that increasing the bsc initially boosts pv_i, this reflects the strategic importance of military strength in supporting territorial expansion to guarantee an appropriate defense of the domains. High bsc implies a more expensive external environment in which only the city-states that plan their expansion properly manage to survive and/or conquer other city-states. Once more is highlighted the crucial role played by the increase of internal productivity to perform all kinds of activities. However, the non-linear growth and subsequent saturation indicate that beyond a certain point, additional investments yield diminishing returns. Despite not being directly predictable by the model structure, the reduction of returns of investing in specific higher preferences is reasonable. Some state variables, such as $ct_i(t)$ or $w_i(t)$ (civil technology and wealth) present exponential growth in costs leading city-states to avoid investing over a certain threshold. A similar behavior can be noted even for the set of preferences where after a certain threshold of cost to perform the activity tends to decrease.

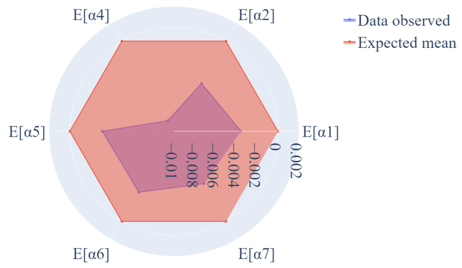
Figure 5d depicts the relationship between bvp and pm_i, clustered by bsc. After reaching a peak, the relationship either plateaus or decreases, suggesting diminishing returns or over-extension in village productivity. These patterns are in line with what has been said so far about the reduction of incentive of excessive level of preferences. With $bsc < t1$ exhibits a rapid rise in pm_i with bvp, peaking around 200 units. Beyond this point, the performance metric plateaus and slightly declines, indicating limited capacity to sustain high production levels with a low recruitment budget. For $bsc > t1$ the graph shows a pronounced increase and a higher peak in pm_i. These clusters maintain higher performance levels for a longer range of bvp, reflecting a balanced approach between village production and recruitment. The decline after the peak is more gradual, indicating that higher bvp can sustain higher bsc better. This conclusion derives from the fact that higher bvp leads to an increase in city-state population that will generate more gold sustaining higher levels of external costs.

When pgp is higher, C_i tend to exhibit higher pv_i and can tolerate higher costs, whether these are allocated for soldier recruitment (bsc) or other expenses. This suggests that C_i with greater economic productivity are better equipped to handle increased operational costs without sacrificing performance or strategic preferences. C_i with higher pgp could support higher pv_i and absorb greater costs. This implies that economically stronger city-states can maintain and even enhance strategic initiatives under higher budgetary pressures. Aggressive strategies might seem to be profitable in the long period not only in the model but also in other scenarios. However, in this analysis, the short period view is not considered and that may generate some differences regarding the results obtained so far. There is a noticeable cost tolerance threshold in both pv_i and pm_i. Beyond certain levels of bsc, preferences decline, indicating that there is a critical balance point in economic and operational planning. This threshold marks the limit at which additional spending on soldier recruitment or other costs no longer yields proportional benefits. The understanding of this balance informs decisions on budget allocations, ensuring that city-states do not overspend on areas that will not contribute to increased performance or strategic advantages.
Higher production levels, both in terms of village production (bvp) and gold production (pgp), positively correlate with higher pv_i. This indicates that productive C_i are more likely to pursue aggressive expansion and strategic initiatives. However, after reaching certain production levels, the incremental benefits of further increases reduce. This suggests that there are optimal production ranges where C_i can maximize their preferences without encountering diminishing returns. Identifying these ranges is key to maintaining efficiency and maximizing the benefits of economic and production activities.
As city-states reach higher levels of production or spending, the additional benefits taper off, indicating that there is a saturation point beyond which further investments do not significantly enhance performance. To counteract diminishing returns, city-states must strategically adjust their investments and focus on maintaining an optimal balance between production, recruitment, and other economic activities.

3.3 In these charts, we compare the expected values of tactical parameters with the actual average values observed in C_i that survived until the end of the simulations. The expected values were calculated based on the range of admissible values by definition. The tactical parameters can take any value in the interval $[-1, 1]$ with equal



(a) Radar plot of the mean values of tactical parameters for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)



(b) Radar plot of the mean values of tactical parameters without $E[a3]$ for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)

Figure 6: Radar plots of tactical parameters

probability distribution. Therefore, their expected value is 0.
The goal is to confront the expected value of a perfectly balanced strategy to the one that in the end, on average, the city-states survived adopted regarding the target selection.
A similar approach was applied to the strategic parameters. The strategic parameters can take any value in the interval $[0, 1]$ with equal probability, but the sum of these parameters must equal 1. For this reason, since there are 8 primary strategic parameters, the expected value for each of them is $1/8$, or 0.125.
Similar to the previous goal, this time the aim is to confront the perfectly balanced set of preferences to the one actually used more frequently by city-states.
Figure 6a illustrate the differences between the expected values of tactical parameters if the environment had no effect on the simulation, and the actual average values obtained from simulations. The tactical parameters show a tendency to be around the expected value of 0 except for α_3, which negative value indicates an aversion on the part of the survived C_i to selecting targets located farther away.
Figure 6b was created by removing $E[\alpha_4]$ from the visualization to better appreciate the differences of the other tactical parameters relative to their expected values. The visualization clearly shows all tactical parameters are slightly negative, indicating a tendency below the expected value of 0. The parameter α_4 remains the most negative among the tactical parameters, underscoring its importance in mission target selection by C_i., highlighting it as the second major contributing factor in determining the target.
The significantly negative value of α_3 suggests a noticeable aversion to selecting distant targets. This may indicate that C_i prefer closer, more manageable targets to optimize their resource allocation and strategic planning. This phenomenon is likely due to the fact that C_i do not have a particularly sophisticated mechanism for controlling armies. This risks putting B_i in a position where they could die from lack of supplies, as they are unable to alter their route to stop and raid a nearby enemy V_i. The observed tactical parameters closely match the expected values, except for α_3. This consistency suggests that, in general, C_i maintain a balanced approach to their tactical initiatives, aligning closely with theoretical expectations.
The slight negativity across all tactical parameters indicates a cautious approach in tactical decisions. This could be due to the inherent risks associated with more aggressive tactics, prompting C_i to adopt a more conservative stance.

Figure 7 shows the average preferences of actions that each C_i can take, comparing them with their expected values. The observed value of $E[pv]$ is markedly higher than the expected mean, indicating that C_i strongly favor expanding through the creation of new V_i. This suggests a strategic preference for growth and expansion as a primary objective. The preference for military actions ($E[pm]$) is slightly above the expected value too. This indicates a secondary focus on military operations, complementing the primary expansion strategy. All the other preferences are below their expected values, so they are less prioritized in comparison to expansion and military actions.
The clear preference of pv_i and pm_i highlights a dominant strategy focused on expansion and military readiness. This expansive strategy seems to perform better in various scenarios within the simulations, as C_i adopting this approach are likely to have survived and thrived. While expansion is emphasised, the slight preference for military actions suggests a balanced approach where C_i not only expand but also maintain a level of military readiness to protect and sustain their growth.

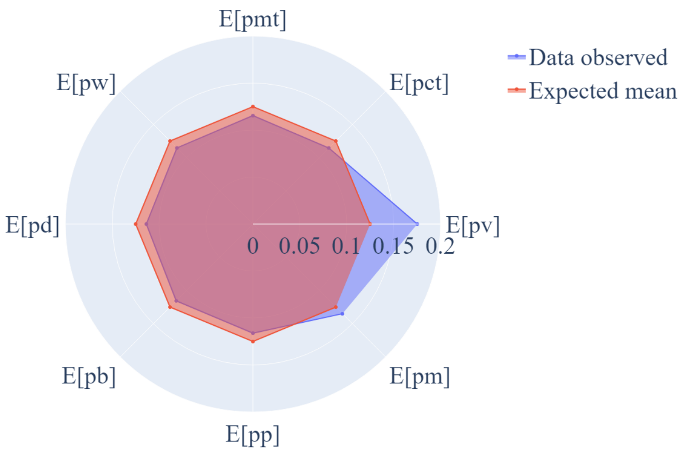
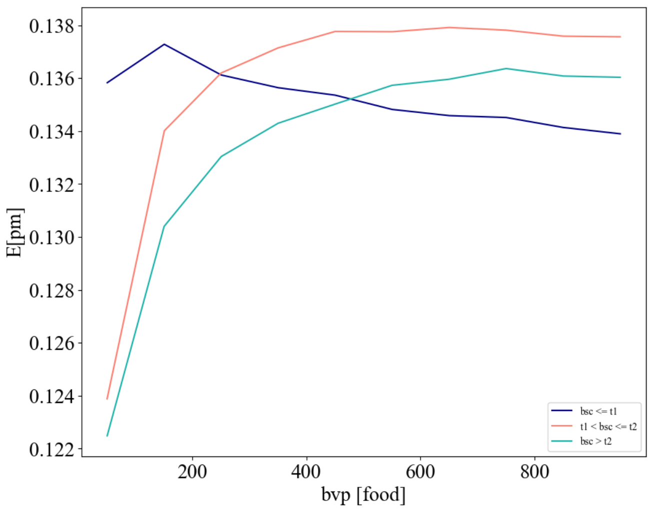


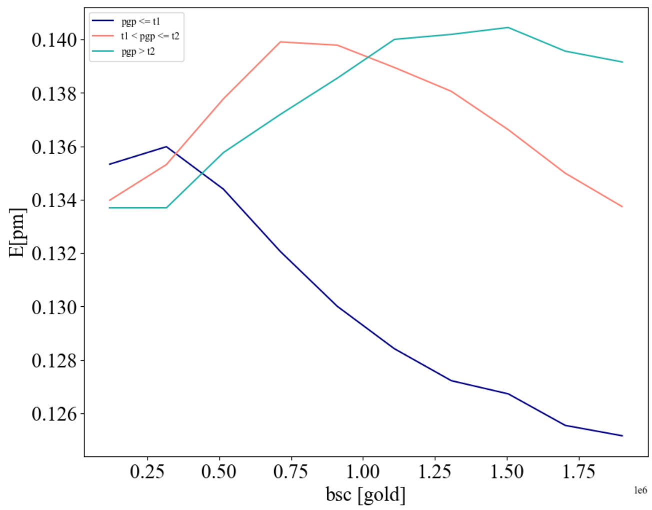
Figure 7: Radar plot of the mean values of strategical parameters for all the C_i lasting in a simulation (in red), compared with the related to expected values (in blue)

3.4 After analyzing the behavior of C_i from an economic perspective, considering the availability of external resources and environmental costs, we will now look deeper into their military behavior. This includes examining their pm_i (preference to organize a military mission) and pb_i (preference to recruit a battalion B_i) in response to varying internal and external parameters. The following analysis, depicted in Figure 8a, Figure 8b, Figure 8c, and Figure 8d, involves plotting a behavioral output on the y-axis, while observing the co-effect of two different inputs: one on the x-axis and the other used to divide the data into three clusters by tertiles, which boundaries are respectively called $t_{var,1}$ and $t_{var,2}$ for each variable. These graphs represent on the y-axis the average values of pm and pb, indicated respectively as $E[pm]$ and $E[pb]$.

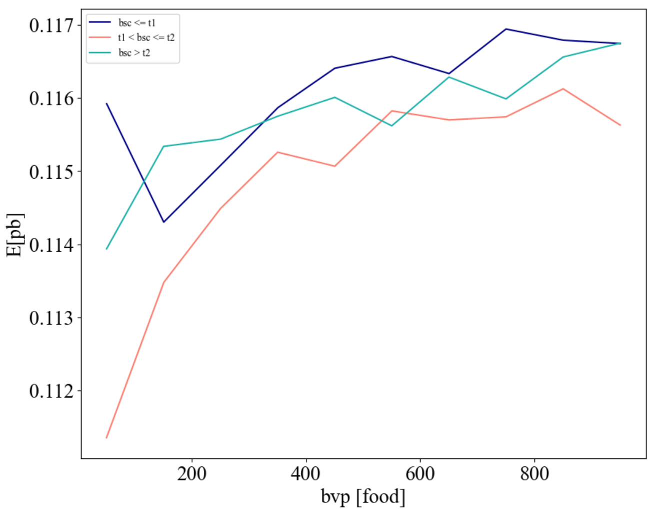
3.5 Figure 8a depicts the relationship between bvp and pm, clustered by bsc. For $bsc < t_1$ the graph starts relatively high, peaks early, and then maintains a fairly stable decreasing trend with slight fluctuations. This indicates that when bsc is low, there is an initial high value of pm_i which stabilizes as bvp increases. For $bsc > t_1$ is shown for both clusters a steep increase initially, followed by a stabilization and slight fluctuations. This suggests that higher bsc lead to a marked increase in pm_i as bvp increases. When bvp is low, C_i are cautious with military missions, especially when bsc are higher. Low bsc can compensate for low productivity to some extent, but there is a threshold beyond which even low costs do not incentivize increased pm_i. Low internal productivity starts a chain reaction where the lack of resources prevents the city-state from investing in wealth $w_i(t)$ or civil technology $ct_i(t)$ without sacrificing the military offense. The pm_i then is reduced to invest more in those activities that can compensate for the poor productivity.
Figure 8b examines the behavior of a city's pm_i in relation to the variation in bsc, used as a reference for the external costs of the environment. The graph includes three distinct curves representing different ranges of pgp as clustering. For low bsc, the pgp is not a significant factor; all three curves converge to very similar values. However, as the costs increase, three distinct patterns emerge. With very low pgp, an increase in bsc results in a significant decline in pm_i. This curve shows a clear negative correlation, indicating a contraction in military strategy as recruitment costs rise, settling at very low pm_i values.
For high pgp, there is a positive correlation between the increase in bsc and pm_i. This suggests that in scenarios where individual battalions hold greater differential value, C_i with high gold production will adopt high pm_i values despite increased bsc.
The intermediate range of pgp is the most interesting pattern, as it combines behaviors seen in both low and high pgp clusters. Initially, there is an increasing trend in pm_i, similar to the high pgp cluster. However, upon reaching a certain threshold, the graph changes concavity and decreases significantly, following the trend of the low pgp cluster until it stabilizes at a value close to the starting point.
The graph suggests that for low bsc, pgp does not significantly affect pm_i. As recruitment costs rise, C_i with low gold production decrease their military activities, those with high gold production increase their activities, and those with intermediate gold production exhibit a two-phase behavior: an initial increase followed by a significant decrease, stabilizing near the initial value. This dynamic pattern suggests a boom-and-bust cycle where after a period of rapid growth and expansion (the boom) it is followed by a sudden contraction or decline (the bust).
Let's now analyze the behavior of C_i in relation to their pb_i concerning variations in internal production param-



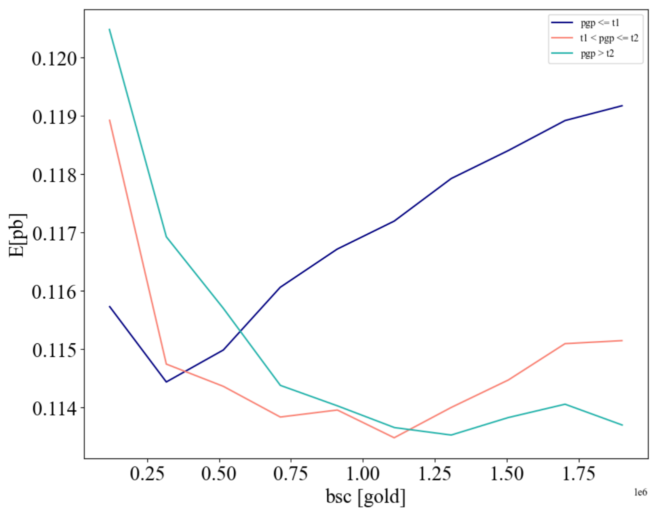
(a) Line plot of $E[pm]$ of C_i related to bvp clustered by bsc



(b) Line plot of $E[pm]$ of C_i related to bsc clustered by pgp



(c) Line plot of $E[pb]$ of C_i related to bvp clustered by bsc



(d) Line plot of $E[pb]$ of C_i related to bsc clustered by pgp

Figure 8: Line plots of $E[pm]$ and $E[pb]$ related to different clusters and x-axis

eters and external costs.
Figure 8c studies the relationship between bvp and pb_i across different ranges of bsc clusters. Overall, the graph demonstrates that asbvp increases, pb_i generally increases across all ranges of bsc. The rate of increase and the initial value vary depending on whether bsc is low, intermediate, or high. For low bsc, there is an initial dip followed by a steady increase in pb_i as bvp increases. This is probably due to the fact that a very low bvp will favorpb_i by those C_i located in more fertile areas, thus partially compensating for the low bvp. By doing so, these C_i will gain a military advantage at the expense of those C_i that cannot afford to feed their soldiers. In this case, therefore, the lowbsc encourages C_i to recruit despite the low level of bvp. As production increases, we notice a dip that brings the graph to values in line with those of the other two clusters, before following an upward trend, consistent with the fact that higher food production allows for the maintenance of a larger number of B_i.
For $bsc > t1$, the trend followed is different. Both clusters start from lower preference values and then follow a steadily increasing logarithmic trend. This trend is justifiable by the fact that a high bsc, associated with a low bvp value, makes it extremely difficult to maintain a large number of B_i in the field, leading C_i to prefer different strategies. However, as bvp increases, the pb_i values rise, with the graph of the higher bsc stabilizing at the same values as the first cluster. This is because, in one case, the low bsc encourages recruitment, while in the other, the high cost makes recruitment difficult, leading C_i to try to gain a differential advantage over their

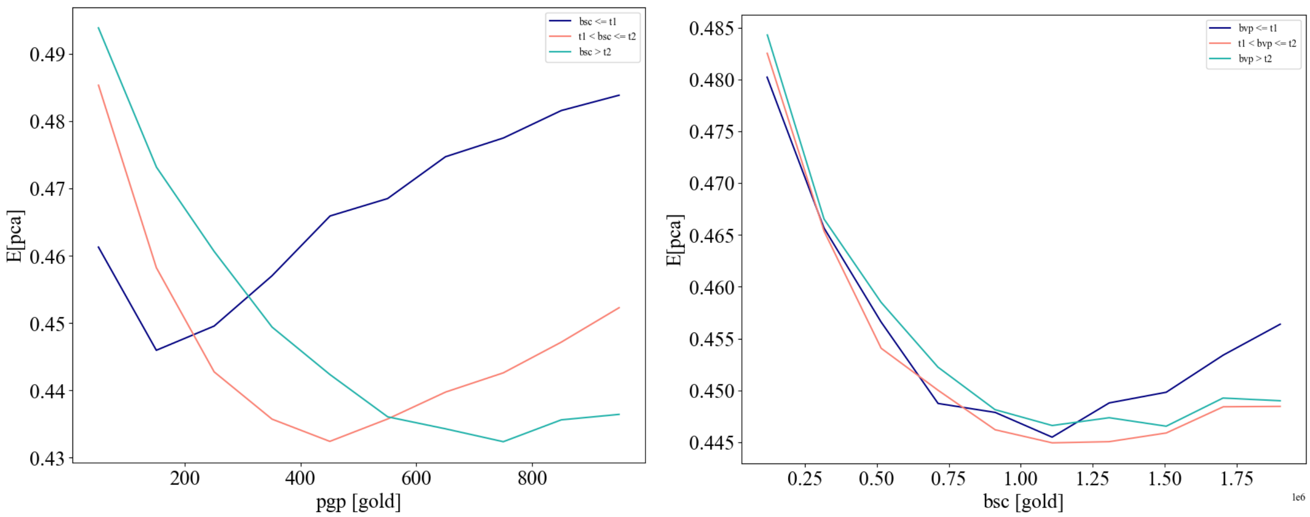
competitors.
This suggests that higher bvp generally encourages a higher pb_i, but the impact is modulated by the level bsc, with higher costs amplifying the positive relationship.
Figure 8d analyze the trend of pb_i in relation to bsc across three clusters determined by the variation of pgp. In the initial phase, the trend observable in the three graphs is very similar. All three clusters start from a more or less high value and then drop drastically with the initial increase in bsc. Subsequently, the graphs adopt diametrically opposite patterns.
For $pgp > t1$, the trend continues to decrease, indicating a reluctance to recruit B_i when the bsc is so high. However, towards the end, there is a slight rise, indicating that for such high costs, the differential value of recruitment is very high.
For $pgp < t1$, after an initial decline, we notice a marked upward trend, reaching significantly higher values than those of the other clusters. Although C_i are able to produce smaller quantities of gold, the pb_i remains very high. This can be explained by the fact that C_i decide to concentrate their limited gold reserves more on recruiting B_i to gain a significant strategic advantage over other C_i and potentially conquer them to appropriate their resources.

From this analysis, it becomes clearer how pm_i correlates positively with city-states production capacity, adopting a more expansive strategy. On the other hand, a lack of resources restricts their propensity for pm_i, preferring to concentrate their activities on other phases.
Regarding the influence of external factors on pm_i, we observe some more interesting patterns. Initially, with low environmental costs, we see an average value of pm_i even for different clusters of city-states' production capacity. However, as costs increase, we notice that a low production capacity of the C_i leads to a drastic decrease in the propensity for pm_i, while particularly high internal production values correspond to the highest pm_i values. In the case of intermediate productivity values, the observed trend is parabolic with a downward concavity, eventually stabilizing at the initial values.
Concerning pb_i, we observe a more predictable trend where an increase in the C_i's production capacity also corresponds to an increase in pb_i. This is clearly due to the city-state's better ability to both recruit and maintain a higher number of B_i. Regarding external environmental influences, internal production capacity plays a differential role in the trend of the graphs. An increase in external costs corresponds to a drastic reduction in pb_i, consistent with expectations, except for an observed trend reversal in the case of particularly low production capacity. As already explained, this is due to the higher differential value that B_i recruitment generates given the city-states' low production.

The analysis of C_i behavior can go into further details by studying the different preferences regarding the type of target for a military mission. This decision involves two diametrically opposed military strategies: on one hand, the objective is to directly conquer the enemy C_i by besieging it and eliminating its defending forces; on the other hand, a more cautious tactic is preferred, aiming to destroy the V_i that supply food to the C_i, thereby starving it, reducing its population, and consequently diminishing its $p_i(t)$ capacity in terms of $g_i(t)$ and B_i.

3.6 These two strategies reflect different approaches to weakening the enemy. Direct conquest focuses on a decisive victory through overwhelming force, while the looting strategy seeks to erode the enemy's strength gradually by cutting off essential resources. The choice between these strategies can depend on various factors, such as the city-states' resources, military strength, and long-term goals. Understanding the conditions under which C_i prefer one strategy over the other can provide deeper insights into their strategic planning and resource management.

3.7 Prior to analysing the two proposed graphs, it is essential to recall that pca_i and pva_i represent two complementary values, $pca_i \wedge pva_i \in R : pca_i \wedge pva_i \in [0, 1] \wedge pca_i + pva_i = 1$. Consequently, an increase in one is accompanied by a decrease in the other, and vice versa. The analysis will focus on the trend of pca_i, with the necessary considerations made. It should be noted that the comments for pva_i would be mirror images. By focusing on the trend of pca_i, the preferences of C_i for direct conquest city-states can be understood. The changes in pca_i under different conditions will shed light on the strategic choices C_i make regarding military missions. For example, a high pca_i value indicates a strong preference for direct city-state conquest, suggesting that C_i are prioritising quick and decisive victories. Conversely, a lower pca_i suggests a preference for a more prolonged strategy of weakening the enemy through attrition by targeting villages V_i. When interpreting the pca_i values, it is important to consider the influence of internal and external factors, such as resource availability, military strength, and environmental conditions. It can be reasonably assumed that a high production capacity may encourage a more aggressive strategy, resulting in a higher pca_i. Conversely, limited resources may encourage a more cautious approach, reducing the pca_i and increasing the pva_i. By understanding these dynamics, it is possible to gain a deeper insight into the strategic considerations that influence whether a C_i chooses direct



(a) Line plot of $E[pm]$ of C_i related to bvp clustered by bsc

(b) Line plot of $E[pm]$ of C_i related to bsc clustered by pgp

confrontation or a strategy of attrition. This analysis will provide insights into the decision-making processes that underpin military strategies in different contextual scenarios. Figure 9a presents a line plot analysis of the trend of pca_i in relation to pgp across three clusters, each defined by a distinct range of bsc. For $bsc < t1$, the initial trend shows a slight decrease in pca_i as pgp increases, followed by a gradual increase. This indicates that, when environmental costs are lower, C_i tend to favour direct conquest as pgp rises. This suggests that when C_i have low external costs, they are more likely to invest in directly conquering enemy C_i as their resources grow. For $t1 < bsc < t2$, pca_i starts at a relatively high value but decreases significantly as pgp increases, reaching a minimum much deeper than the previous cluster. After this dip, pca_i starts to increase again. This pattern indicates that for C_i, when environmental costs are moderate, the preference for direct conquest initially decreases as they invest more in accumulating gold. However, once a certain level of resources is guaranteed, they shift back towards direct conquest. For$bsc > t2$, pca_i decreases steadily as pgp increases, reaching its lowest point. Then pca_i starts to increase slightly but remains relatively stable. This indicates that when environmental costs are high, C_i initially reduce their preference for direct conquest as they increase gold production. However, as their resources grow further, they maintain a low but steady preference for direct conquest. This analysis demonstrates how C_i' strategic preferences for direct conquest (pca_i) are influenced by their internal production capacities (pgp) and external costs (bsc). Those with lower environmental costs are more likely to pursue direct conquest with increasing resources, while those with higher costs tend to adopt a more cautious approach, potentially focusing on other strategies such as attrition (pva_i).

Figure 9b presents a line plot analysis of the trend of pca_i in relation to bsc across three clusters, defined by different ranges of bvp. The graph illustrates that the strategic choice of C_i to attack enemy C_i is only marginally influenced by their ability to produce food. All three graphs exhibit a strikingly similar pattern, initially declining and then rising again towards the end. For $bvp < t1$, pca_i starts at a relatively elevated value and declines steadily as bsc increases, reaching its lowest point. Subsequently, the value of pca_i begins to increase, indicating that C_i with low internal food production values tend to reduce their preference for direct conquest as environmental costs increase. However, this trend reverses at higher levels of bsc. For $bvp > t1$, the graph indicates that even C_i with high internal production values initially reduce their preference for direct conquest with rising environmental costs, but then show a modest increase afterwards. This suggests that, across all clusters, there is a clear initial decrease in pca_i with rising environmental costs, which may be taken to indicate that higher external costs generally discourage direct conquest strategies.

Discussion and conclusions

4.1 The results of this study shed light on the dynamic interplay between economic and military strategies within city-state systems under resource constraints. Through an agent-based modeling approach, the paper provided insights into how city-states manage the balance of growth, resource management, and survival in a competitive environment.

The first key finding is the significant role of economic productivity rate in shaping surviving city-states' strategic

		City-state Productivity		Environment Costs	
		High	**Low**	**High**	**Low**
Economy	**Pv**	Expansive	Conservative	Expansive	Conservative
Strategic Military	**Pb**	Expansive	Conservative	Divergent	Expansive
	Pm	Expansive	Conservative	Divergent	Converging
Tactical Military	**Pca**	Reducing	Increasing	Reducing	Increasing
	Pva	Increasing	Reducing	Increasing	Reducing

Figure 10: Summary table of city-states C_i behaviour

preferences. Higher economic output, particularly in terms of gold and food production, tends to support more aggressive expansion and military strategies. This is evident from the positive relationship between production rates (pgp and bvp), preferences for both village founding (pv_i), and military mission organization (pm_i). The capacity to generate resources allows city-states to sustain larger military forces and undertake more ambitious actions, without putting at risk their survival.

Conversely, limited resources impose stringent constraints on city-states, forcing them to adopt more conservative strategies to last until the end of the simulation. The results show that when faced with high external costs (such as bsc), city-states with lower economic productivity reduce their military activities and shift focus towards survival and resource conservation. This finding underscores the importance of economic strength in maintaining strategic flexibility and resilience.

The tactical decision-making of city-states also highlights interesting patterns. The preference for direct city-state assaults (pca_i) versus indirect strategies like targeting villages (pva_i) varies with resource availability and environmental conditions. City-states with higher gold production (pgp) tend to favor direct assaults as their resources increase, reflecting a strategy aimed at achieving decisive victories. In contrast, high environmental costs (such as bsc) generally discourage direct assaults, prompting city-states to adopt more cautious approaches that avoid over extension and potential depletion of resources.

Moreover, the observed non-linearities and diminishing returns in strategic preferences suggest that city-states must carefully calibrate their investments. The initial phases of resource allocation often yield significant benefits, but beyond certain thresholds, additional investments result in diminishing returns. This phenomenon is particularly evident in the trends of village founding (pv_i) and military mission organization (pm_i), where initial increases plateau or even decline after reaching specific levels of production or cost.

The study's findings on the clustering effects of different strategic parameters further elucidate the complexity of decision-making in competitive environments. For instance, city-states in an environment with high number of starting city-states (N) display a greater propensity for expansion and military actions, suggesting that a larger base of C_i provides a buffer against resource scarcity and external threats. This aligns with real-world scenarios where demographic strength often underpins economic and military capabilities.

The summary table presented in Figure 10 provides an overview of the city-state's strategic approach, categorising it as either expansive or conservative. In the context of tactical military strategy, the terms "Reducing" and "Increasing" were employed due to their complementary nature. The term "Divergent" is used to indicate a differentiation of behaviour according to the internal city-state productivity, whereas the term "Converging" implies a marginal effect by the internal productivity. The table is structured to facilitate an intuitive visualisation of economic and military performance. As previously discussed, the high productivity of a city-state should theoretically correspond with low environmental costs. However, this is not always the case, as evidenced by observations in the economic and tactical military domains. (Wang et al. 2012; Cil & Mala 2010; Epstein 1999)

For future developments of the model, one could explore using, for example, a genetic algorithm to find the best possible strategy within the pool of numerous combinations available.

Also, the model could be extended incorporating additional variables and more complex interactions to en-

hance its relevance and generalizability. For example, a memory could be added to agents, to allow them to remember past interactions, conflicts, and trade agreements; this could lead to more sophisticated decision-making processes and to the usage of historical data to predict resource scarcity, adding a layer of strategic planning and foresight, or expliciting consider the reputation of other city-states according to their past behaviour.
Another possible model extension is the addiction of trade, to create a more interconnected and interdependent system, permitting city-states to compensate for their weaknesses by leveraging the strengths of others, and introducing the possibility of more cooperative interactions. Also, including the possibility to form alliances could significantly alter the dynamics of the model, even including multilateral interaction. Finally, various elements such as internal politics in city-states decision making, environmental changes that affect food production, specific geographical features such as mountains and seas that could affect the success of a city-states

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