A Gravity Model for Emergency Departments

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ABSTRACT

The issue of facility location is of significant importance in numerous systems, where the efficient utilisation of resources is of great importance. Gravity models, which are inspired by Newtonian physics, are commonly employed to address these problems and have a long tradition of being used in healthcare. The objective of this paper is to enhance the comprehension of patients' decision-making processes in emergency healthcare by introducing an extension to existing gravity models, which includes two novel factors influencing emergency department choice: hospital sizes and patients' severity. The newly formulated gravity rule, which integrates these factors, demonstrated an extremely high accuracy against real-world data in terms of overall hospital location and flows between cities and hospitals, respectively 98.77% and 98.02%.

Please note: Abbreviations should be introduced at the first mention in the main text – no abbreviations lists. Suggested structure of main text (not enforced) is provided below.

1 Introduction

The facility location problems are relevant across various systems due to their critical role in the efficient utilization of resources (1). Usually, these problems can be solved by determining the better locations for a set of facilities in a given solution space, in such a way that maximizes users' accessibility and minimizes operational costs, thereby ensuring effective resource allocation (2). This decision-making process is crucial in different sectors such as logistics (3), urban planning (4), healthcare (5; 6), and, in general, in every context where the facility placement affects the transportation costs, the level of services, and the operational effectiveness (7).

Various methodologies have been developed to address location problems, each offering specific advantages (8). For example, linear programming is widely employed for solving location problems due to its effectiveness in handling large-scale linear models (9). However, it cannot be always employed when the problem structure does not permit the representation of real-world features with linear functions (9). Metaheuristic optimization presents a more flexible alternative (10) since this method is not confined to linear constraints and can be efficiently employed when the solution spaces are complex (11), at the cost of a much higher computational effort (12). Also, qualitative methodologies such as Delphi methods have been employed (1).

A noteworthy specific case of the facility location problem arises when one or more locations should serve a large number of users distributed in a given space (13; 14). To effectively undertake such challenges, a possible approach involves leveraging the assumption that the potential customers' likelihood of visiting a facility is inversely proportional to their distance from it (15), mirroring the Newton's law of universal attraction. The models that employ this metaphor are aptly termed 'gravity models' (16), and can include other non-physics related features (17).

Gravity models have been employed to deal with different problems, such as the interpretation of the trade flow between two nations using GDPs as function of inter-country distances (18), the location of new logistics hubs (19), and the inference of retail facility attractiveness from secondary data regarding customers' buying power and sales volumes (20). In healthcare, gravity models have also been utilized for many years (21; 22), offering insights for the decision-making process and policy formulation, with interesting potentialities for hospitals' improvement and application in various healthcare services' settings (23; 24; 25; 26; 27).

This paper endeavours to enhance the current understanding of the patients' decision-making process in emergency healthcare scenarios, improving the state-of-the-art related to gravity models (17; 26; 28; 29). The contribution to the field regards the way two new factors, in addition to distance, influence the choice of emergency care department (30): the hospital size and the patients' perceived severity. The model relies on the assumption that a larger hospital, presumably with more extensive facilities and resources, is more likely to be the preferred choice for patients (31), and that this preference is amplified by the patients' perceived severity of their condition. Especially, the more critical the condition, the greater the likelihood of a patient opting for a larger hospital. To the best of our knowledge, this hypothesis has never been investigated before, but it could be extremely relevant to better understand patients' behaviour and improve emergency departments facility locations.

The novel gravity rule underwent a calibration and validation phase on real-world data from emergency care units from Lombardy (Italy) (32), to gain relevancy to policy-makers (33; 34), which yielded results that surpassed current benchmarks in the field. The outcome of this empirical testing shows that the mean error between real-world data and the results simulated with the gravity model is approximately 1.23%.

2 Research objects and methods

This section presents the proposed gravity model and its calibration to a specific geographical area, firstly providing a formal description of our gravity meta-model for hospital selection, detailing the foundational modeling hypotheses, the variables incorporated into the model, and their interplay; then the application of this model in a specific context is illustratedi including a thorough explanation of the methodologies and procedures employed in our experiments, ensuring scalability and generalizability of our findings. The Python notebook utilized for data analysis, along with a representative subset of the data used, are publicly available.

2.1 Gravity meta-model

This research focuses on developing a function that accurately model the patient likelihood of choosing a particular hospital, considering distance and various other determinants. Unlike existing models that often rely on general preference data, this approach is event-based specifically targets the final stage of the decision-making process (35). This stage captures the definitive choice a patient makes, either independently or through their transportation medium, in cases where the patient is not autonomous. Thus, the structure of the candidate class of functions is as follows:

$$p_k^*(H_i) = f(d_{k,i}, t, b_i) \tag{1}$$

where $p_k^*(H_i)$ stands for the supposed preference of a patient k to choose the hospital H_i among all the N hospitals available. d_i is the distance between patient k and the hospital i, t_k the triage code upon arrival, which represents the severity of patient k, and b_i the average perceived size of hospital i, serving as a proxy for its emergency care department accessibility, which is the same for every patient k.

The distance $d_{k,i}$ is computed using the public API from the Open Source Routing Machine using Python 3.9 on the 13th September, 2022. The metric was selected under the hypothesis that the preference towards a specific hospital is affected by the duration of the travel and not by the actual routing distance. Thus, the traveling time between every city in the area of analysis was collected, measured in seconds. Also, since it is not possible to gain precise information regarding the exact place where a patient k starts, it was hypothesised that, for area inhabitants, the residence town is a sufficiently good approximation of the town where they are located when they decided to which hospital to go (36). The triage code t_k is assigned to each patient k at the entrance of an emergency care department and embodies the evaluation of patients' severity made by nurses when they arrives at the emergency department of reference. The Italian healthcare system categorizes the severity of a patient's condition using color codes, in ascending order of severity: white, green, yellow, and red, which are respectively encoded in a non-ordinal numerical variable whose value can be 1, 2, 3, and 4. A further code, the black one, also exists to include patients that are deceased upon arrival at the emergency department. This model considers the triage code as a proxy for the seriousness of the condition of a patient at the moment they leave their residence, not including the black codes in the analysis. This classification was later updated, but at the time of data collection a four levels scale was adopted. The parameter b_i stands for the size perceived by a patient regarding a specific hospital H_i during the decision, given that data regarding individual preferences are not available. The rationale is that a hospital with more beds could be considered to have a "higher" quality than a hospital with fewer beds (37). We use the term "perceived" because patients, in a situation of bounded rationality, are not supposed to know the exact services provided by each hospital; even so, they do not have the time and the resources to process this information. Also, even if a survey were conducted, the results could be affected by two elements. First, the answer would not be given in a moment of stress, such as the one in which a patient is choosing which emergency care department to direct to. Second, it would not include the decision-making of rescue vehicle drivers, which should be obtained by means of a second survey. Since the goal of this work is to find a universal and simple rule that can simulate the distribution of individuals in different areas, we used as a proxy of the perceived accessibility parameter the number of beds in the emergency care department of the hospital H_i. This metric takes into consideration both the perception of private individuals and rescue vehicle drivers, since all of them are affected in their location selection process by the size of the hospital, and it has also high adherence to reality, since the dimension of the hospital is likely connected to the number of healthcare services that it is able to offer, representing a direct measure of the accessibility to the healthcare system (38).

Given these hypotheses, the following preference function is proposed.

$$p_k^*(H_i) = \frac{b_i^{\alpha t_k}}{d_{k,i}^{\beta}} \tag{2}$$

The function combines b_i, t, and d_i by raising the perceived accessibility to the power of t and dividing it by the distance. α and β regulate the effect of each factor. This equation implies that as the perceived accessibility increases or the severity of the patient's condition intensifies (higher b_i and t values), the preference for a particular hospital $p^*(H_i)$ also increases. Conversely, there is an inverse relationship between the distance from a hospital $d_{k,i}$ and the preference toward it, as that the longer it takes to get to the hospital, the less likely it is for a patient to direct there.

From all the N preferences $p^*(H_i)$, a probability for a patient to head to the H_i structure can be derived as

$$p_k(H_i) = \frac{p_k^*(H_i)}{\sum_{i=1}^{N} p_k^*(H_i)} \tag{3}$$

Maintaining the metaphor of gravity, the probability for a patient to go to the hospital H_i in a given condition is proportional to the attractiveness of H_i.

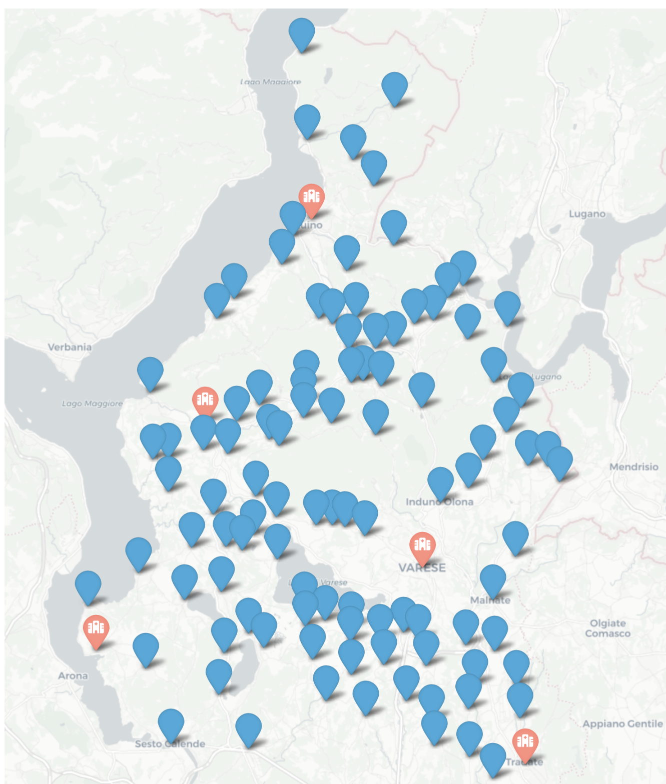


Figure 1. Geographical position of the towns in the considered area, divided for town where no emergency departments are present (red marker) or absent (blue marker)

2.2 Model specification

The meta-model presented in the previous paragraph was adapted to a specific area, the region served by the hospitals associated with the Sette Laghi Territorial Social and Healthcare Organizations (ASST), a healthcare facility located in the northern part of Lombardy in Italy. Figure 2 depicts the geographical distribution of cities in the area and the cities where an emergency care department is present. Specifically, this study focuses on six hospital facilities, each with a corresponding emergency department, which provides comprehensive medical aids to the population, including specialized treatments, diagnostic services, and emergency care. The ASST includes six different emergency departments, summarized in Table 1, with the number of beds that have been used to calibrate the model. One could argue that the patients' perception regarding a hospital do not depend only on the size of the emergency care department, but also from the overall number of beds of the whole hospital; so, Table 1 also reports the total number of beds. Since they strongly linearly correlates, the number of beds in the emergency care department was employed in the calibration and validation, as it was more in scope with the purpose of the model.

The dataset utilized in this study comprises patient behavior records in the specified region, encompassing patient arrivals at an emergency department during 2019, the second half of 2021, and the first half of 2022. This dataset documents a total of $325,886$ arrivals. Each entry details relevant information, such as the severity level and the patients' city of residence. For the purpose of this analysis, patients not residing in towns within the ASST Sette Laghi area were excluded, resulting in a consideration of $256,701$ emergency department arrivals.

The process of model calibration involves determining the optimal values of parameters α and β to minimize the discrepancy between the observed and simulated distributions of resident arrivals at emergency departments within the study area. The simulation of the distribution is executed as follows: a random subset of $25,000$ records, each representing a patient arrival at an emergency unit, is extracted from the dataset. For each record, a selection probability $p(H_i)$ is calculated for each hospital H_i. Subsequently, a destination hospital H_i^* is chosen based on this probability distribution, and the patient is assigned to that hospital. Upon allocating all $25,000$ patients, a specific metric is utilized to evaluate the divergence between the actual and simulated distributions for each hospital.

$$e_i = |f_i^r - f_i^s| \tag{4}$$

where f_i^r and f_i^s respectively the relative frequency of H_i for real data and simulated data. A genetic algorithm is then employed to estimate α and β minimizing $max(e_i)$. The algorithm runs for 250 generations, with a population of $5,000$ possible solutions.

3 Results

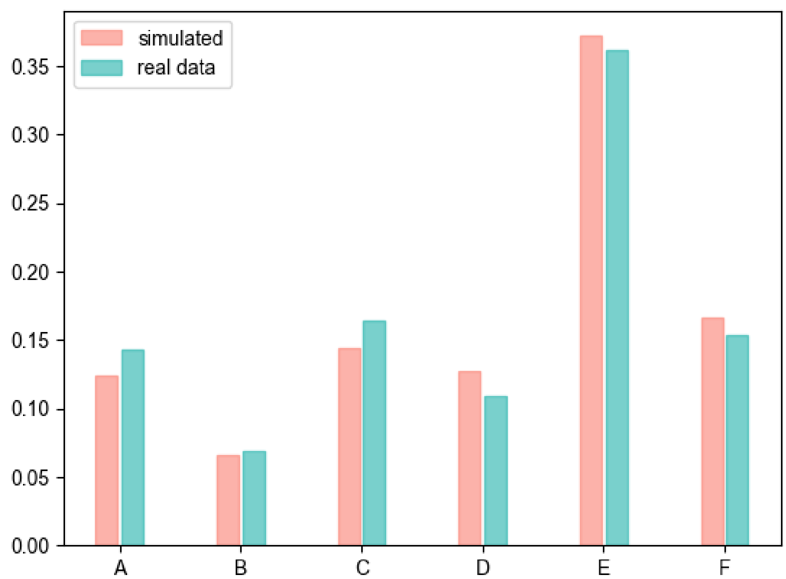
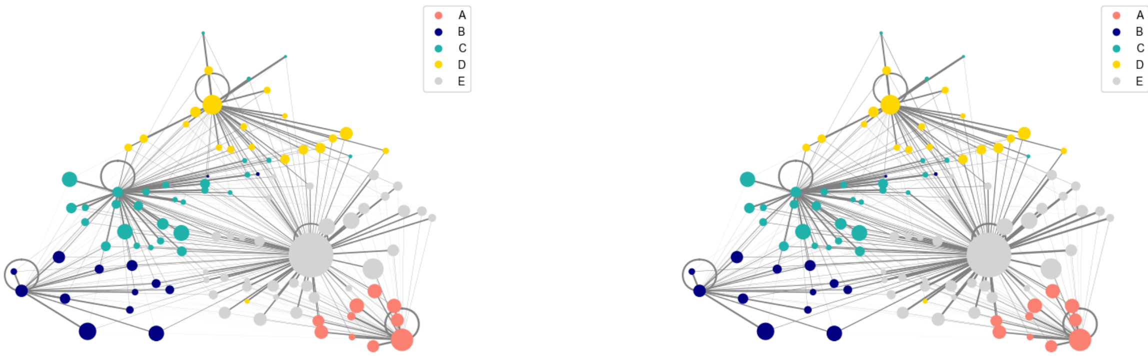


Figure 2. Comparative analysis of the relative distributions of patient arrivals at each hospital level

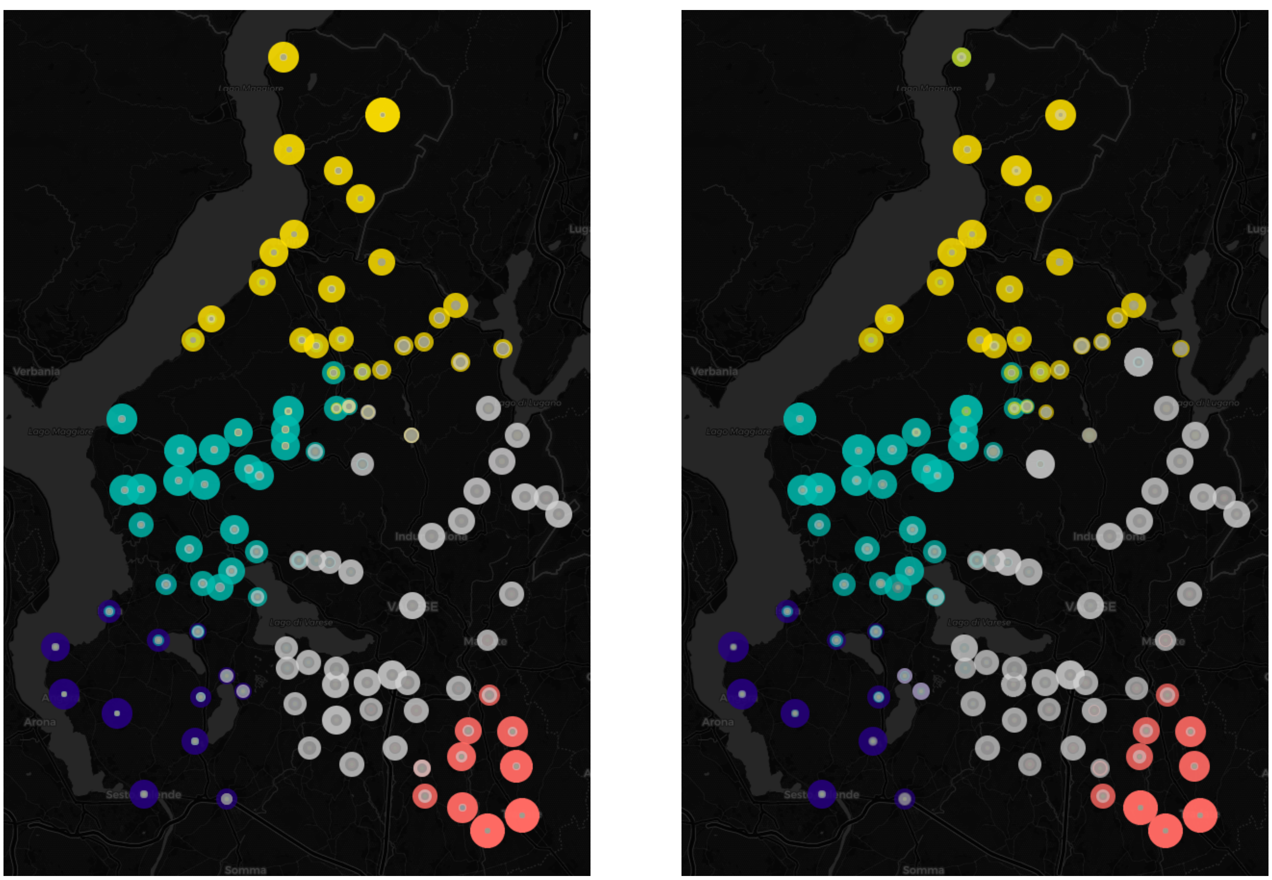
The results divide into three distinct parts. The first encompasses an aggregate analysis, which underscores the efficacy of the model in accurately allocating the population to the appropriate emergency care departments, while the second focuses on how this allocation is achieved, emphasizing the retention of the unique characteristics inherent in the allocation network. Finally, the patient allocation differences between real-world data and simulated data is shown for each city. This threefold approach was adopted to provide the most comprehensive understanding of the proposed methodology performance and its precision with a progressively higher level of detail.



(a) Real data **(b)** Simulated data

Figure 3. Comparison of the network of distributions between real data and the results of the simulated data. The color stands from the targeted city in which most of the population has gone, while the size embody the population of the corresponding city.

Figure Figure 2 displays the relative frequency of allocating the population from a city to a hospital, as delineated in the model specification section. The experimental results indicate that the maximum discrepancy between any two elements in the relative distribution is $max(e_i) = 0.0215$, while the average error is quantified at $E[e_i] = 0.0123$. This implies that the calibrated model can assign each patient, based on their triage code and city of residence, to the appropriate hospital with an average precision of 0.9877.



(a) Real data **(b)** Simulated data

Figure 4. Share of the patients heading to a given hospital per each city

Figure 3 presents a network visualization of this complex system (39; 40), where each node represents a city, positioned according to its geographical coordinates (longitude and latitude). Each link denotes a movement of inhabitants from the residency city x to an emergency department in another city y where an hospital H_i is located. The color of each node indicates the city of the emergency department to which the majority of a city's population travels, while the node size corresponds to the population size of the city. The comparative analysis of these networks provides two observations. Firstly, employing travel time as a metric, as opposed to Euclidean distance or travel distance, preserves geographical proximity, confirming the proposed model's adherence to real-world spatial relationships. Secondly, it is evident how the model not only accurately allocates individuals to hospitals but also precisely reconstructs the network of connections between cities of residence and hospital locations (41). This dual capability of the model highlights its effectiveness in both individual allocation and in mapping the broader network of healthcare access.

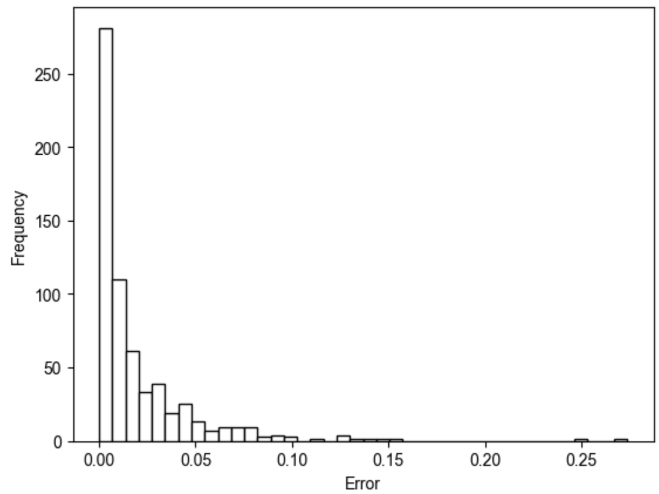


Figure 5. Distribution of the error of evaluation for each combination of city of origin an city of the hospital

To enhance the reliability of the results and add a quantitative perspective to Figure 3, a final analysis was conducted to assess the precision of the model's allocation of each city's population to a hospital. This analysis evaluates the proportion of individuals from a residency city x attending a hospital i in both real data and the outcomes predicted by the gravity model. Figure 4 shows the comparison of the proportion of patients traveling from a specific town to a given hospital. The sub-figures distinguish between real and simulated data, while the color code is consistent with that used in Figure 3. The graphs suggest that, with some minor variation, the share of patients traveling from a residency city x to a hospital i can be accurately reproduced by the model.

Finally, we quantify the allocation error for each residency city-destination hospital pair as $e_{x,i}$, the distribution of which is illustrated in Figure 5. Although the distribution is right-skewed, indicating some higher values, the average error across all cities is 1.982%, and the median error is 0.897%. Nevertheless, the majority of the errors come from small towns. For towns with a population above $7,000$ inhabitants, which is the threshold for city classification in Italy, the mean error is only 0.699%.

4 Discussion

The gravity model introduced in this paper demonstrates precision in two key aspects: the specific allocation of individuals to hospitals, which is executed with high accuracy (as illustrated in Figure 2), and in determining the contribution of cities, denoted as j, to specific hospitals, labelled as i, in terms of patient flow (see Figure 3 and Figure 5).

Common sense and previous studies suggest that the workload of a hospital is influenced by the population size and the availability of alternative healthcare options in the vicinity (21; 29). This gravity model provides a means to account for these factors, suggesting how factors different than physical proximity, such as the attractiveness of an hospital, could help to better fit the data (25). Usually, these fine-tuning operations usually complicate the model, and even when they have a great fit with data, they become less explainable with every new assumption (42). The main merit of this model is to explain a complex behaviour with a very simple rule (43), employing simplifications such as the absence of personal networks (44).

Drawing from the previous outcomes, discernible spatial patterns become evident concerning the possible geographical accessibility to emergency care departments within the specified territories examined in the analysis, when the conditions of patients are taken into account. One significant inquiry arises: what are the underlying reasons for the distinctive spatial pattern observed in terms of potential access to emergency departments? (23) While factors such as topography and historical settlement patterns have certainly influenced the current distribution of the population (35), it is evident that these spatial patterns cannot be solely attributed to them. Another crucial explanatory factor is the decision-making process in healthcare services, which has decided both the position of the emergency department and the assignation of a fourfold progressive code to classify the severity of a patient's condition. These systemic factors are likely to contribute to the spatial patterns identified through the modified gravity model. Nevertheless, their precise role remains uncertain at present. To address this knowledge gap, it would be necessary to engage in consultations with decision-makers responsible for resource allocation, system design, and other key functions (23).

5 Conclusion

This paper aims to advance the understanding of patient decision-making in emergency healthcare by refining gravity models, focusing on how hospital size and patient condition severity, beyond just distance, influence emergency department choice. A new gravity rule, surpassing current benchmarks, was developed and tested, showing a mean error of 1.23% when applied to real-world data, highlighting its precision and potential in predictive modelling.

Even if yielding promising results, this study is subject to certain limitations that merit consideration. Firstly, there is the need for further validation in diverse geographical areas, to validate the model with more diverse data (45). Designed with generality in mind, the model is theoretically applicable to any region where Geographic Information System data are available and patient arrivals at the emergency department are encoded into a hierarchical classification system. However, the current research does not assert that the relationships identified are universally applicable across different contexts or areas, and further empirical studies are needed to assess the model's adaptability and efficacy in other settings.

Another notable limitation pertains to the distinction between patients' residential locations and the actual departure point. The present model solely accounts for the former, lacking data on the latter. This absence of information about the actual starting location of emergency care journeys may limit the accuracy of the model, even thought the results are very promising. The gravity model demonstrates robust results when analyzing patient allocation based on residential areas, but a more precise fit to the data might remain impracticable without the integration of additional information. It highlights the need for an even more comprehensive data collection.

In light of the findings of this study, several avenues for future research have been identified to further substantiate and expand upon the current results. First, replicating the current analysis across a broader and more diverse range of territories could validate the results obtained in this study and test the applicability and robustness of this gravity model extension in different geographic and demographic contexts, increasing the scientific relevance of the work (46). Also, further analysis could assess whether the patterns and relationships identified are consistent across various settings or depends on specific geographical features (47). Second, developing a simulation model could additionally confirm the results of this study (48). For example, an agent-based model simulating the interactions of individual patients within a healthcare system could offer a powerful tool for testing hypotheses and observing emergent behaviours under varying conditions (49). Finally, causal inference model of micro individual behaviour aimed at directly confirming the relationship between individual decision-making processes and the variables identified in this study as influential factors in patient behaviour (50) might be designed.

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Author contributions statement

E.F. secured the founding and the data access. F.S and F.A. collected and pre-processed the data. F.B. analyzed the data, conceived the experiment, conducted the experiment, and analysed the results. F.B. and F.S. wrote the first draft of the manuscript. All authors reviewed the manuscript.

Data availability statements

The code and a subsection of the data used for the experiments are available in the following repository: https://anonymous.4open.science/r/grav submission. The total data are available under reasonable requests.

Additional information

The authors do not have any competing interests.

Figure legends

- Figure 1: geographical position of the towns in the considered area, divided for town where no emergency departments are present (red marker) or absent (blue marker);
- Figure 2: comparative analysis of the relative distributions of patient arrivals at each hospital level;
- Figure 3: comparison of the network of distributions between real data and the results of the simulated data. The color stands from the targeted city in which most of the population has gone, while the size embody the population of the corresponding city;
- Figure 4: share of the patients heading to a given hospital per each city;
- Figure 5: distribution of the error of evaluation for each combination of city of origin an city of the hospital

Tables

Hospital code	City	# Total beds	# beds in ED
1	A	112	25
2	B	40	10
3	C	86	15
4	D	44	13
5	E	469	84
6	E	139	15

Table 1. Hospitals with emergency department in the ASST Sette Laghi, with the number of beds places in the hospital