An improved gravity model for Emergency Departments: from patients' preferences to healthcare operations' efficiency

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Abstract

This study investigates patients' decision-making process, focusing on Emergency Departments (EDs) selection, also considering the creation of information supporting the healthcare planning. An extension of the gravity model, integrating factors such as the distance between patients' home and EDs, the hospital's size and the clinical conditions' severity is proposed. The model is applied to a specific geographical context related to a University Hospital composed of six territorial facilities and located in the Northern Italy, highlighting a model accuracy equal to 98.7%.
The study provides both theoretical and practical contributions, for the mathematical modelling and for supporting the healthcare operations' efficiency.

Keywords: gravity model, emergency departments, facility location

Purpose

Facility location problems are relevant for a variety of sectors to minimize operational costs, while ensuring an effective resources' utilization. In healthcare context, they are typically related to the need of ensuring an adequate level of accessibility for patients while minimizing locations' cost. In this sector, characterized by a growing demand of services, scarce resources, healthcare professionals' shortage, patients' pathway fragmentation among several hospitals but also territorial healthcare facilities, the ability of predicting patients' flows and hospitals' admissions is becoming a crucial and pivotal challenge (Zhou et al., 2023). For these reasons, the mathematical modelling is very useful and widely applied in healthcare facility location problems (Earnshaw and Dennett, 2003) to support the operations' planning and definition.

Focusing on the Emergency Departments, in particular, some common problems hamper Emergency Departments' operations such as overcrowding, patients' long length of stay, staff shortages, arrival volume increases, and budget constraints (Collins, 2021). These problems could lead to a higher workload, a decrease in the patient's satisfaction in combination with longer waiting times and delayed inpatient admissions (Gross et al., 2023; Collins, 2021; Skowron et al., 2019), considering the high number of patients accessing the EDs and the difficulty of the system to take in charge all the patients and to ensure an appropriate capacity to deliver healthcare services and procedures.

Extended waiting and service times concur to the quality-of-care reduction, also increasing the risk of adverse events and negative outcomes for the patients. Thus, being characterized by high uncertainties and the need for quick decisions under the above-detailed constraints and criticisms, EDs presents several challenges for operations management. Minimizing resource consumption is at the basis for the implementation of efficient healthcare services and, for this reason, several studies provide significant evidence of alternative methods, procedures, or tools to improve performance, ranging from prediction tools (Chen et al., 2022), to the development of new instruments and predictive tools (Webb and Mills, 2019; Saghafian et al., 2014).
In this field, gravity models, traditionally developed and implemented for retail and marketing issues (Reilly, 1953), or transportation problems (Hallefjord and Joernsten, 1986), are applied to offer important and significant insights for the decision-making process. Nevertheless, the existing literature evidence indicates a low accuracy rate and overlooks certain significant aspects, that may impact on the selection of an ED, such as the hospital size, the patients' perceived severity, and the distance between the patient's residence and the healthcare facilities location (Rogelj and Bogataj, 2018).

Focusing on the healthcare emergency management, this study investigates the patients' decision-making processes proposing an extension and a different implementation of the classic gravity model (Cu˜nat and Zymek, 2022; Yuk et al., 2020), adding the above-mentioned factors to the traditional mathematical modelling, increasing the knowledge of patients' decision-making processes and contributing to the gravity models' theoretical field.

Theoretical background: gravity models in healthcare

The gravity models in healthcare are applied in the literature as a well-established tool to capture the interplay of several variables that could influence patients' flow, and consequently hospitals' admissions (Congdon, 2010; Mayhew, 1984). Many authors considered the interactions between the geographic factors (Flowerdew & Aitkin, 1982) with the information related to the outpatient and inpatient accesses (Xiong et al., 2016).

In addition, these mathematical models, also integrating the Bayesian approaches to define more precise predictions (Congdon, 2000), specifically of the patients' flows with reference to the resources' allocation and planning among the different hospital' departments and units (Teow et al., 2017; de Mello-Sampayo, 2014).
Some studies demonstrated the efficacy of the gravity models not only predicting the general hospitalizations but focusing also on specific types of admissions such as the nursery or the intensive care unit (Latruwe et al., 2022; Lowe & Sen, 2006) and different territorial settings such as the rural areas (Wende et al., 2020).

Table 1 reports a summary of the already published evidence related to gravity models define in the scientific literature and applied within the healthcare sector, considering that only one study is focused on Emergency units or Emergency Departments as organizational settings of reference for the model.

Author(s)	**Year**	**Title**	**Geographical context**	**Organisational setting**	**Aim/objective**	**Parameters**
Lowe and Sen	2006	Gravity model applications in health planning: Analysis of an urban hospital market	Chicago	Hospital	Forecast change in patients' flow due to proposed change in hospital financing policies	Patients' access, hospital success, travel time, number of travels. patient's category (health insurance or not)
Congdon	2010	A Bayesian approach to prediction using the gravity model, with an application to patient flow modeling	United Kingdom	Emergency Departments and units	Estimate and predict the hospitalization flows by Bayesian methods of hospitalization	Demand, distance from patient's home to hospital, distance-based accessibility
Crooks and Shuurman	2012	Interpreting the results of a modified gravity model: examining access to primary health care physicians in five Canadian provinces and territories	Canada	Primary care	Understand how potential spatial access to primay health care physicians can be interpreted in some Canadian jurisdictions using a modified gravity model	Supply in terms of physicians' number, demand, travel time
de Mello-Sampayo	2014	Gravity for health: an application to state mental hospital admissions in Texas	Texas	Mental care units	Model patient flows from the residential regions to health suppliers' regions	Average length of stay, forensic services, travel distance adjacency, a competition factor

Xiong et al.	2016	Using the fusion proximal area method and gravity method to identify areas with physician shortages	Shangai (China)	Territorial area	Integrate different methods for measuring spatial access to health resources within a large area, in order, in particular, to find geographical areas characterized by physicians' shortage	Supply capacity, distance or travel time, total number of supply locations, total number of demand locations
Teow et al.	2017	Applying gravity model to predict demand of public hospital beds	Singapore	Hospital	Estimate the demand of the new hospital to plan infrastructure and manpower using Gravity model analysis	Patient days of existing hospitals, patients' residence, gravity mass coefficient of each hospital
Wende	2020	Planning health care capacities with a gravity equation	Germania	Hospital and rural areas	Planning health capacities and ensuring adequate access to health care	Size of the population, distance between the population and healthcare providers, providers' attractiveness ,variations in the number of service providers and their tariffs
Latruwe et al.	2022	Improving inpatient and daycare admission estimates with gravity models	Belgium	Hospital	Improve the methods at the disposal of policymakers and hospital administrators to optimize hospital facility location decisions, considering, in particular, the patients' choice for the inpatient and daycare hospital accesses	Proximity hospital size, accessibility
Shi	2022	Spatial accessibility patterns to public hospitals in shanghai: an improved gravity model	China	Hospital	Assess the spatial accessibility patterns to public hospitals in Shanghai through point density analysis and a comprehensively improved gravity model	Class capacity of hospitals, service ability, travel time, population quantity and travel impedance

Table 1 – Summary of Gravity Model evidence applied in the healthcare sector and available in the literature.

Methodology

From a methodological perspective, the study is articulated into two different steps: firstly, a formal description of the proposed gravity model, presenting the modelling hypotheses, the variables included in the model and the relationship occurring between the variables themselves; then, an empirical case is proposed applying the model in a specific geographical area related to a University Hospital composed of different territorial facilities.

Focusing on the development of a function able to accurately model the patient likelihood of choosing a particular hospital, considering distance and other determinants, a function with the following structure is defined:

$$p^*(H_i) = f\,(d_i, t, q_i) \tag{1}$$

where $p^*(H_i)$ is the individual patient preference to choose the hospital H_i among all the n available hospitals, d_i the distance between the patient and the hospital H_i (computed using public API from the Open Source Routing Machine in Python), t_k the triage code (which stands for the pathology's seriousness) assigned by a nurse to a patient at the arrival in an ED, and q_i, the average perceived quality of hospital i, which in this work is treated using the hospital's size as a proxy, assuming that a hospital with a higher number of beds may be considered as to have a "higher" quality than a hospital with fewer beds.

Specifically, the following preference function is proposed to describe individual preferences regarding the hospital H_i:

$$p^*(H_i)= \frac{q_i^{\alpha t}}{d_i^{\beta}} \tag{2}$$

where α and β are two parameters that regulate the effect of the distance and of the perceived hospital's quality. Here, the preference regarding a structure grows with its perceived quality, and this effect is moderated by the patient's condition intensifies (higher q and t values); oppositely, the higher the distance from a hospital, the lower the preference toward it.

The proposed model is tested in a specific context, using real-world data collected between 2019, 2021 and 2022 from the six EDs related to the territorial facilities of the Sette Laghi Hospital, a teaching, University organisation in the Northern Italy and adopting a dataset, concerning 325,886 EDs accesses, that provides information about patients' residence and severity level. A simulation approach is adopted to calibrate the model and determining the optimal values for the parameters α and β in order to minimize the difference between the observed and the simulated values.

Findings

Consistent results were achieved implementing the model, considering both the efficiency of the model in allocating the patients to the appropriate EDs and the approach of the model to compute the allocation process.

Figure 1 shows both the relative frequency of allocating the population to a specific hospital (considering the six EDs of the Sette Laghi Hospital organization, numbered from 0 to 5) and the real data, obtaining an average error of $E[e_i]=0.0123$ and implying a precision of the model equal to 98.7% in assigning each patient to the appropriate ED, considering the priority code and the city of residence.

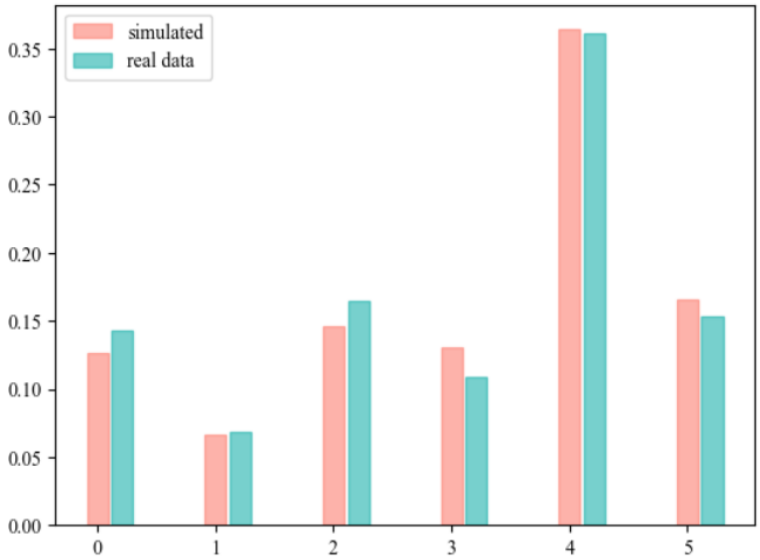

Figure 1 – Comparative analysis of the relative distribution of patients' arrivals at each territorial facility of Sette Laghi Hospital (numbered from 0 to 5).

Considering the system's network visualization, in which each node represents a city and each link a movement from city *j* to an ED situated in another city *k,* the following comparative analysis is defined (Figure 2).

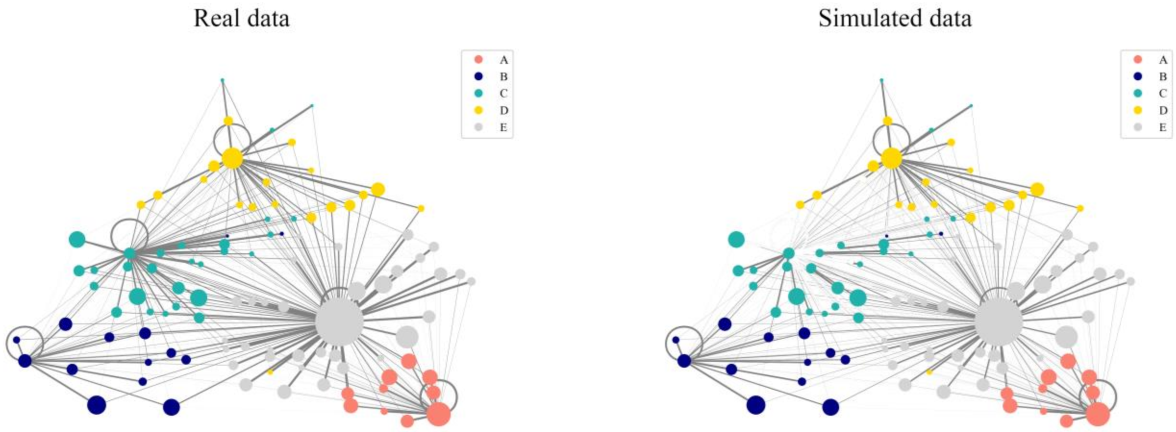

Figure 2 – Comparison of the network considering the real data and the results of the simulation.

In Figure 2, the color of each node indicates the ED's city picked by the majority of a city's population travels, and the node size corresponds to population. The figure depicts the capacity of the model to consider the geographical proximity (adhering to the real-world spatial relationships), allocating accurately patients to EDs, maintaining the connections between residence cities and EDs' locations.

Discussion and conclusions

The results are consistent with previous evidence, where a gravity model is employed to facilitate healthcare capacity planning (Wende et al., 2020; Tao et al., 2018), assuming that the volume of population and the presence of other healthcare facilities and emergency points in a specific geographical area could affect and influence the patients' choice and, as a consequence, also the workload and the operations of an ED (Teow et al., 2017).

The present study has several implications, both from a theoretical and a practical/managerial perspective. Firstly, the study could more generally enlarge the research field of mathematical modelling, answering to previous critical issues related to other tools and models already implemented for the EDs' organisational settings. An example could be the advanced multi-objective optimization models such as the one provided by El-Sawy and colleagues (2022), that incorporates patient's length of stay and waiting time within machine learning techniques, explored to assist the EDs operations planning by predicting waiting and treatment times, allowing for better management of workload and resources' allocation (van Delft, 2022).
In addition to these approaches, gravity models present a promising avenue for enhancing EDs operations by predicting patient inflows and thus, anticipating patient arrivals more accurately due to a more complete understanding of patient flow patterns, which is crucial for strategic planning and resources' distribution, positively impacting on the accessibility of delivery services. The incorporation of gravity models into EDs operational planning frameworks and models could further optimize patient care pathways, both from the hospitals and the patients' perspectives. Human and structural resources could be optimized and allocated efficiently, with a precise hospital planning. Patients could receive timely medical services and assistance, with a higher level of appropriateness, and at the same time, creating more accessibility to the EDs procedures.

From a practical point of view, the main contribution is related to the possibility of understanding the causal relationships between a patient's characteristics, the peculiarities of a healthcare facility (in this case an Emergency Department, but the model could be extended in general to all the healthcare services or facilities) and the patient's decision on which facility visits. The adoption of this conceptual model aims to explore and quantify the influence of various factors on patient choice, thus offering valuable insights into the optimisation of the healthcare services' distribution, within a specific context and the related accessibility. Moreover, the study could potentially support the healthcare operations within the Emergency Departments, but also the services' sizing, in terms of scheduling, involved medical specialisation and workforce planning, improving the overall quality and efficiency, both for the healthcare professionals and the patients (Garg et al., 2010). In addition, these findings are becoming pivotal considering the need to integrate territorial facilities within the current healthcare ecosystem, primarily focused in most of the cases on the hospital facilities, as suggested by the Italian National Decree n.77 and the National Recovering and Resilience Plan, supporting the definition of possible patients' cases that could be taken in charge and treated also in other healthcare facilities, more capillary distributed in a specific geographical area.

Furthermore, this study is subject to some limitations that may be considered in the discussion phase. Firstly, the proposed model is tested only in one geographic context in the Northern Italy and future geographical areas could be analysed and considered to ensure the reliability and accuracy, assuming that the model could work in all the context in which Geographic Information System (GIS) data is available to map the travel between patients' home and Emergency Departments. In addition, the decision of patients located in a metropolitan area must be further investigated, to confirm the model's generalizability and accuracy, because they could vary having more choices for the same distance covered.

Another notable limitation pertains to the distinction between patients' residence and the actual departure point (that could be different from the residence location, included into the database), affecting the model's accuracy.

Future research should be devoted to implement a simulation approach to model the interactions between the patients (representing the agents) and the healthcare system facilities (assumed as the environment of reference), potentially confirming the results of this study and observing emerging behaviours in terms of patients' choice and network configuration. In addition, the model could be applied also to the territorial facilities' accesses, supporting the development of an innovative healthcare network with a patient-centred approach. Furthermore, from a theoretical perspective, the model should be applied to other real data to validate the reliability and the accuracy aspects.

In conclusion, the present study would enable ED managers and healthcare professionals to monitor ED processes with a dynamic mathematical tool, supporting the resources management in an efficient way, and implementing optimal strategies for handling overcrowding and critical issues.

References

Chen, W., Argon, N. T., Bohrmann, T., Linthicum, B., Lopiano, K., Mehrotra, A., ... & Ziya, S. (2023). Using hospital admission predictions at triage for improving patient length of stay in emergency departments. Operations Research, 71(5), 1733-1755.

Collins, J. (2021). Improving Emergency Department Throughput: Using a Pull Method of Patient Flow.

Congdon, P., 2001. The development of gravity models for hospital patient flows under system change: A bayesian modelling approach. Health Care Management Science 4, 289–304

Congdon, P., 2010. A bayesian approach to prediction using the gravity model, with an application to patient flow modeling. Geographical Analysis 32, 205–224. doi:10.1111/J.1538-4632.2000.TB00425.X.

Cu˜nat, A., Zymek, R., 2022. The (structural) gravity of epidemics. CESifo Working Paper Series doi:10.2139/ssrn.3603830.

de Mello-Sampayo, F., 2014. Gravity for health: an application to state mental hospital admissions in texas. Munich Personal RePEc Archive URL: https://mpra.ub.uni-muenchen.de/59758/.

Earnshaw, S., Dennett, S.L., 2003. Integer/linear mathematical programming models: a tool for allocating healthcare resources. PharmacoEconomics 21 12, 839–51.

El-Sawy, N. M., Ali, D. S., Saleh, M., & Mohamed, O. M. S. (2022, November). Improving the Emergency Department by Using Simulation and Optimization. In International Conference on Advanced Intelligent Systems and Informatics (pp. 395-409). Cham: Springer International Publishing.

Flowerdew, R., Aitkin, M., 1982. A method of fitting the gravity model based on the poisson distribution. Journal of regional science 22 2, 191–202. doi:10.1111/J.1467-9787.1982.TB00744.X.

Garg, L., McClean, S., Meenan, B., Millard, P., 2010. A non-homogeneous discrete time markov model for admission scheduling and resource planning in a cost or capacity constrained healthcare system. Health Care Management Science 13, 155–169. doi:10.1007/S10729-009-9120-0.

Gross, T. K., Lane, N. E., Timm, N. L., Conners, G. P., Hoffmann, J., Hsu, B., ... & COMMITTEE ON PEDIATRIC EMERGENCY MEDICINE. (2023). Crowding in the emergency department: challenges and best practices for the care of children. Pediatrics, 151(3).

Hallefjord, Å., & Jörnsten, K. (1986). Gravity models with multiple objectives—theory and applications. Transportation Research Part B: Methodological, 20(1), 19-39.

Latruwe, T., Van der Wee, M., Vanleenhove, P., Michielsen, K., Verbrugge, S., Colle, D., 2022. Improving inpatient and daycare admission estimates with gravity models. Health Services and Outcomes Research Methodology, 1–16.

Lowe, J.M., Sen, A., 2006. Gravity model applications in health planning: Analysis of an urban hospital market. Journal of Regional Science 36, 437–461. doi:10.1111/J.1467-9787.1996.TB01111.X.

Mayhew, L., 1984. Gravity modelling: Application to health systems, 1004–1008doi:10.1007/978-3-642-69939-9232.

Reilly, W. J. (1953). The law of retail gravitation. (No Title).

Rogelj, V., Bogataj, D., 2018. Planning the home and facility-based care dynamics using the multiple decrement approach: The case study for slovenia. IFAC-PapersOnLine 51, 1004–1009.

Saghafian, S., Hopp, W. J., Van Oyen, M. P., Desmond, J. S., & Kronick, S. L. (2014). Complexity-augmented triage: A tool for improving patient safety and operational efficiency. Manufacturing & Service Operations Management, 16(3), 329-345.

Skowron, N., Wilke, P., Bernhard, M., Hegerl, U., & Gries, A. (2019). Workload in emergency departments: a problem for personnel and patients?. Der Anaesthesist, 68, 762-769.

Tao, Z., Zheng, Q., Kong, H., 2018. A modified gravity p-median model for optimizing facility locations. Journal of Systems Science and Information 6, 421 – 434. doi:10.21078/JSSI-2018-421-14.

Teow, K.L., Tan, K.B., Phua, H.P., Zhecheng, Z., 2017. Applying gravity model to predict demand of public hospital beds. Operations Research for Health Care doi:10.1016/j.orhc.2017.09.006

van Delft, R. A., & de Carvalho, R. M. (2022). Using Machine Learning Techniques to Support the Emergency Department. Computing and Informatics, 41(1), 154-171.

Webb, E. M., & Mills, A. F. (2019). Incentive–compatible prehospital triage in emergency medical services. Production and Operations Management, 28(9), 2221-2241.

Wende, D., Kopetsch, T., Richter, W.F., 2020. Planning health care capacities with a gravity equation. 888, Ruhr Economic Papers.

Xiong, X., Jin, C., Chen, H., Luo, L., 2016. Using the fusion proximal area method and gravity method to identify areas with physician shortages. PLOS ONE 11, e0163504.

Yuk, S., Hong, B., Song, J., Hwang, I.S., Cha, G., Lee, S., 2020. A study on the force and center of gravity of the transfer-lift for the human stability of spine patients doi:10.18178/ijmerr.9.4.612-617.

Zhou, J., Brent, A. J., Clifton, D. A., Walker, A. S., & Eyre, D. W. (2023). Improving patient flow through hospitals with machine learning based discharge prediction. medRxiv, 2023-05.