Emergence of Risk Sensitivity from First Principles: A Multi-Agent-Based model

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Abstract. Risk preferences have attracted a large amount of debate in different disciplines. Here, we propose a possible explanation for the origin of risk sensitivity based on the supposition that they can emerge in complex systems from genetic and cultural evolution. A multi-agent-based model was developed to test the plausibility of this hypothesis. In the model, agents survive choosing between a riskier and a safer decision. The contribution of this paper is threefold. First, it provides an overview of risk sensitivity emergence in a population of agents. Second, it appraises the effect of environmental factors on this process, and it analyzes the influence of adaptation style on that effect. Finally, it highlights non-monotonic and non-linear relationships between the emerging risk preferences and the dangerousness of the in-silico environment. The simplicity of the proposed approach suggests that these findings could apply to different application fields.

Keywords: risk sensitivity, risk preferences, multi-agent-based model, emergence

1 Introduction

Risk aversion and risk-seeking are recognized characteristics of human [28, 15] and non-human behaviour. Several works show how various kinds of entities respond to risk. In biology, there are proofs of risk-sensible action in non-human species such as animals [11], plants [13] and bacteria [5]. Risk preferences also appear at a super-human level, such as in organizations [2]. Whatever is the level of analysis, the risk sensitivity of living entities seems to be shaped by the same common fundamental principles. All the considered disciplines assume that the chance of taking a risky decision is affected by the variability of the possible outcomes of the choice [15]. Individuals that favour choice with low uncertainty are considered risk aversed. When the stakes are adequately high, individuals prefer the safer option. The similarity of these features suggests that it could exist a regularity regarding how living entities develop risk-sensible behaviour.

Several works investigate this instance. In-silico experiments show risk aversion can emerge when individuals interact in a risky environment under evolutive pressure

[26, 8, 30, 17, 15, 9]. Pieces of evidence from cognitive neuroscience support the hypothesis of the evolutionary origin of risk aversion. Nevertheless, some suggest embracing a wider perspective, including the role of cultural processes [25, 3]. Various disciplines deal with entities and populations that adapt risk preferences by learning [18]. Also, proofs of concept show that risk preferences can emergence through reinforcement learning [21]. The adaptation style, which is the way a population of entities adjust to different conditions, is especially relevant in systems composed of entities that both learn and are selected by their fitness to the environment. These systems include ecosystems as well as financial markets. There is also a lack of studies toward the effect of environmental variables on the emergence of risk preferences. Szpiro highlights a non-monotonic relationship between the harshness of the environment and the risk aversion of the population [26]. Hintze et al. indicate that risk aversion emerges from an evolutionary process in the equivalent mean payoff gamble only when a risk is a rare event with a large impact on the individual's fitness [15]. A multidisciplinary review of the principal risk preferences theories finds an agreement around the existence of an effect of environmental factors on the emergence of risk-sensitive decision making in human and non-human animals [20].

This work aims to confirm that risk sensitivity can emerge in a population from spatially explicit first principles individuals' interactions, described in the following section. Nevertheless, its principal purpose is to assess the effect of environmental factors on this emergent phenomenon, and how the adaptation style affects this relationship. A multi-agent-based modeling simulation (MABS) technique was applied to address the issues. MABS is a technique for modelling and simulating dynamical systems as an assortment of autonomous decision-making entities [6]. It permits to include a difference between actual risk and individuals' risk identification [19]. While MABS is not the solely available methodology [22], evidence suggests it fits this specific problem. Risk sensitivity appears in a set of biological and socio-economical systems considered complex [17], which means collections of heterogeneous agents interacting, co-adapting and co-evolving [24]. MABS is well suited for dealing with these entities [12, 27]. MABS is also a suitable tool to investigate emergent phenomena [4]. Our goal here is to observe a specific kind of emergence at a population level, which is (at least) partially dependent on the underlying process of interaction of single individuals. This work presents a simple agent-based model in which a single breed of agents lives into a two-dimensional artificial world. Agents have two receptors and can choose between two options: supply energy from a passive energy source or attempt to steal energy from another agent. The first is safe, while the second implies the possibility to fail. In this paper, we regard risk sensitivity as the difference between risk proneness and risk aversion of agents. We called it a "first principles" model because it relies on these two simple rules without any other assumptions. It follows an analogue works on the evolution of cooperation [10].

The model simulations show that risk aversion and risk-seeking behaviours emerge from the interaction of agents. A relationship between environmental factors and the emergence of risk sensitivity is found. This relationship seems to be affected by the adaptation style. Moreover, it seems to exist a non-monotonic relationship between environmental dangerousness and the emergence of risk sensitivity. The simplicity of this MABS hints that these results could apply to various application fields. It is coherent with the claims for generalization of previous works [26, 15, 9].

The rest of the paper proceeds as follows. In Section 2, we overview the first-principles agent-based model. Section 3 reviews the experimental setup. Section 4 presents the simulation results from the model. In Section 5, the results are discussed. Lastly, we display in Section 6 our conclusions and further work.

2 Model

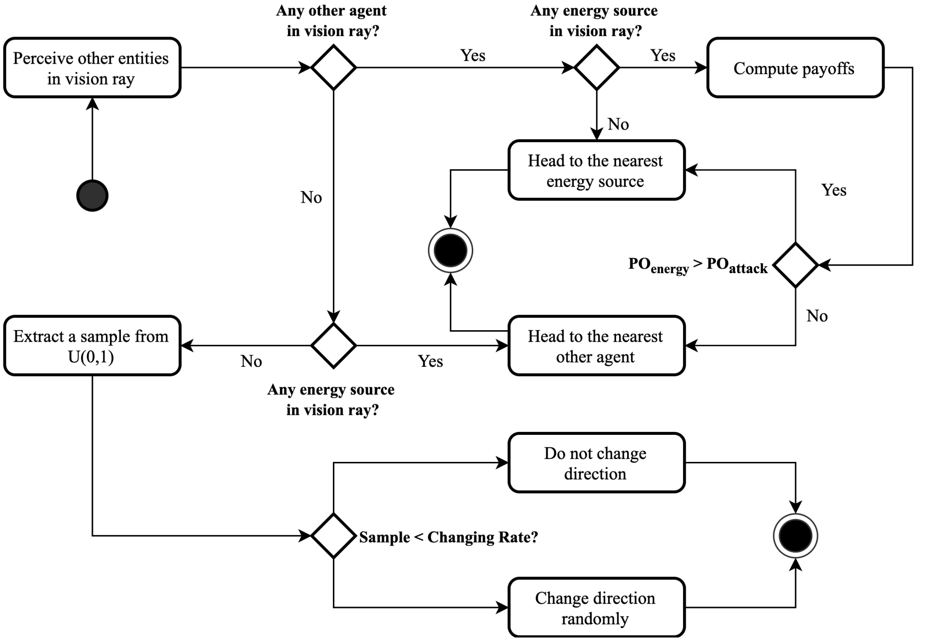
We introduce here the model underlying the simulation results, displayed in the following section. The landscape is a two-dimensional toroidal surface divided into squared cells. Each entity is located on a cell. Thus, the model is spatially explicit [12]. Cells either contain an energy source or are empty. Sources have an initial level of energy, progressively drained by agents. Once an energy spring is completely consumed, it disappears, and another appears in a different cell. Agents consume a given amount of energy at each time step. When an agent runs out of energy, it dies. Therefore, agents need to recharge to survive. There are two ways to do it. An agent can point to an energy source. An agent can move one cell per tick, and only to a von Neumann neighbourhood. When the agent arrives on the cell of the energy source, it can supply from it. Otherwise, an agent can direct toward another agent and attack it. Aggressions have a probability of success. A victorious agent subtracts a share of energy from the defending agent. Oppositely, a losing one gives a part of its energy to the defending agent. The purpose of the model is to make agents decide between these two options. One is risky (to attack), and one is safe (to supply to an energy source). For each possibility, agents compute a payoff, which formula is the distance from the option i times the desirability of that opportunity,

$$PO_i = \left(\frac{d_i}{v}\right)[l_i \cdot lrn\_w + g_i \cdot (1 - lrn\_w)] \tag{1}$$

where d_i is the distance of an agent from the option i, v the maximum distance of perception, l_i the desirability of option i learned by experience, g_i the generic desirability of option i, and lrn_w adaptation style of the population, which stands for the overall preference between a decision making driven by genetic adaptation or by cultural adaptation.

The nearer the target, the higher the payoff. For $lrn\_w = 0$, decision making of agents is influenced solely by genetic adaptation. For $lrn\_w = 1$, it is determined entirely by experience. For intermediate values, decision making is a mix. The higher α_i, the greater the relative influence of previous events on the decision-making. Agents select the choice with the highest payoff. When only one option is available (i.e., in vision ray), agents pick it. When no options are available, agents go straight or change direction randomly. Figure 1 shows the decision making of agents.

Fig. 1. Decision making of agents in a flowchart



Risk sensitivity of an agent is the difference between risk proneness and risk aversion, weighted for the adaptation style. Mathematically, we define this as,

$$rs = (l_{RA} - l_{RS}) \cdot lrn\_w + (g_{RA} - g_{RS}) \cdot (1 - lrn\_w) \tag{2}$$

At the beginning of each simulation each agent has l_i equal to 0.5. g_i is sampled from a continuous uniform random distribution, which bounds are 0 and 1. Agents breed through sexual reproduction, with a given reproduction rate. Every generation event produces one agent. At each event, an agent picks a partner randomly. The genotype of the offspring is the random combination of the genotype of its parents. An offspring can inherit a certain amount of memory from its parents. The amount of experience inherited corresponds to the level of cultural transmission. Genes do not vary during the lifetime, while the information about the world is updated by learning. When an agent chooses one of the two options, it assesses its consequence with regards to energy gain and memorize it. The sum of its experiences constitutes the judgment about the expected result of a possibility, which is the variable l_i. Agents have limited memory and can remember only up to a certain number of events. When the memory is full and a new event is appraised, the oldest is forgotten to make space.

3 Experimental design

The model was implemented on NetLogo 6.2.0[1], a programmable modelling environment specifically designed for MABS [29]. The simulations were employed on two Windows machines equipped with a 3.30GHz Intel(R) Core(TM) i5-4590 CPU and 4.0 GB RAM. We performed a global sensitivity analysis to appraise the environmental effect on the output. Sensitivity analysis examines how the uncertainty in the output can be apportioned to distinct model inputs [23]. A sensitivity analysis is global when this effect is estimated by varying together all parameters, and not only appraising small perturbations near an input space value. We did not have any preliminary assumptions about values and causal relationships while developing the model, so an analysis of the whole set of parameters was necessary. Model parameters were mixed employing uniform probability distributions to denote the uncertainty and obtain a comprehensive outlook of the space state. The parameters analyzed are shown in Table 1, with the relative sampling distributions. We considered a single simulation output, the mean risk sensitivity of the population. Results include only simulations in which the population survived until the end, if not specified differently. The model was simulated 300000 times, and each simulation lasted 4000 ticks. We treated and analyzed the output data with Python 3.8, in a Jupyter notebook.

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[1] The source available at Github, https://github.com/submission20210301/MABS2021.

Table 1. Parameters mixed in the global sensitivity analysis

Parameters	**Min**	**Max**	**Kind**	**Unit**	**Description**
w_size	15	35	discrete	cells	side of toroidal surface
v	1	5	continuous	cells	maximum vision distance of agents
a_max	100	300	discrete	# agents	maximum number of agents
l_avg	50	1000	discrete	ticks	average life duration of agents
rr	0.01	0.1	continuous	adimensional	reproduction rate of agents
e_num	0	0.1	discrete	# sources	number of energy sources per patch
e_lev	0	10	continuous	energy	initial level of energy for each
e_max	2	10	continuous	energy	maximum level of energy stored by
e_in	0.2	2	continuous	energy / tick	maximum level of energy supply
e_con	0	0.25	continuous	energy / tick	energy consumed by agents
mm	1	100	discrete	ticks	maximum weight length of memory
lrn_w	0	1	continuous	adimensional	of experience in decision-making
lrn_t	0	1	continuous	adimensional	% memory transmitted to offsprings
att_c	0	1	continuous	adimensional	chance that an attack is successful
att_g	0	1	continuous	adimensional	% of energy stolen when prevail
att_l	0	1	continuous	adimensional	% of energy lost when defeated

4 Simulation Results

This section is divided into three parts:

1. an overview of how risk preferences arise from the simulation of the model,
2. a prove of a relationship between the risk sensitivity in a population of agents and the simulated environment, and how it is affected by the adaptation style,
3. an outline of a connection between the dangerousness of the environment and the emergence of risk sensitivity.

4.1 Emergence of risk sensitivity

Results on the compete parameter space showed that risk sensitivity emerged in the simulated model. Figure 1 illustrates that both risk-averse and risk-seeking behaviour arose from the simulation of the model. Also, a certain number of runs remained risk-neutral. What is more, the mean of the distribution was 0.2349. It means that the majority of simulated agents became risk aversed through adaptation.

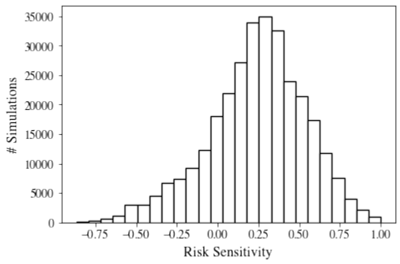

Fig. 2. Distribution of risk sensitivity on surviving populations

4.2 Effect of simulation parameters on risk sensitivity

In this section, we have a twofold goal. Identifying the relationship between the risk sensitivity emergence process and environmental factors, and assessing the influence of adaptation style on this connection. A way to address this issue was to measure the variation of parameters for simulation in which agents develop different risk preferences. Mathematically, we define this as,

$$s = \mu_{RA} - \mu_{RS}$$

were μ_{RA} is the normalized mean of a parameter for the simulation in which risk aversion emerged, and μ_{RS} is the normalized mean of a parameter for the simulation in which the population of agents became risk loving. For each parameter, two scenarios were analysed. An evolution scenario (ES), which considered only simulations with $lrn\_w < 0.1$ (i.e., populations which mainly adapt through evolution). A learning scenario (LS), including solely runs with $lrn\_w > 0.9$ (variation technique driven by learning). As specified in Section 3, $lrn\_w \in (0,1) \subset \mathbb{R}$. Table 2 shows that in most cases differences between the mean values of the parameters were significative. Notable exceptions were the maximum number of agents when there is not only evolutive adaptation, the reproduction rate under cultural adaptation, and the energy stored per tick rate under cultural adaptation. It meant that environmental factors affected significantly risk sensitivity emergence direction. Moreover, some parameters had a positive value in the ES scenario and negative in LS scenario, and vice versa. In Section 5 we discuss this result.

Table 2. Difference of normalized mean values of each parameter for simulations with risk averse population and simulation with risk prone population, for different adaptation styles

Parameter	LS		ES	
	d	P-value	d	P-value
w_size	-0.1309	0.0000	-0.0293	0.0019
v	0.2262	0.0000	-0.0714	0.0000
a_max	-0.0045	0.4504	0.0893	0.0000
l_avg	0.0632	0.0000	-0.0380	0.0000
rr	-0.0056	0.3633	-0.0575	0.0000
e_num	-0.1437	0.0000	-0.2286	0.0000
e_lev	-0.2116	0.0000	-0.1637	0.0000
e_max	0.0626	0.0000	-0.0789	0.0000
e_in	-0.0682	0.0000	0.0065	0.4735
e_cons	-0.1384	0.0000	-0.2246	0.0000
mm	-0.1206	0.0000	0.0037	0.7043
att_c	0.4753	0.0000	0.0876	0.0000
att_g	0.1864	0.0000	0.0168	0.0669
att_l	-0.1125	0.0000	0.0797	0.0000

4.3 Relationship between dangerousness of environment and emergence of risk sensitivity

To enable viewing the results in a 2D plot, we needed to restrict the analysis to pairs of variables. The harshness of a parameter configuration can be gathered by the rate of simulations in which the population do not survive. We obtained a dangerousness index through an ordinary linear regression, to estimate how parameters in Table 1 affected this survival rate. Thus, the higher the dangerousness, the lower the chance for the population to survive. The relationship between risk sensitivity and dangerousness was studied with regards to the adaptation style, as in Figure 3. From left to right, the weight of experience on the decision-making process of the agents is increasing. It is possible to observe that the shapes of the curves change with the adaptation style. In the following section, these findings are discussed.

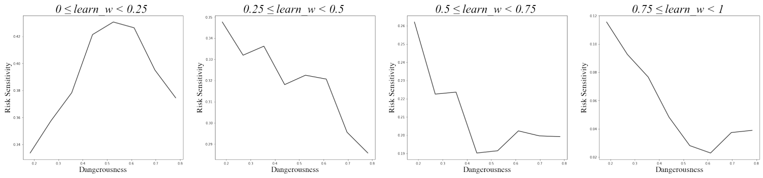

Fig. 3. Risk sensitivity of population plotted on the dangerousness of the system with respect to different style of adaptation, from evolution (left) to learning (right)

5 Discussion

This paragraph briefly examines the theoretical implication of simulation results, namely, how they contribute to the debate relating to the emergence of risk preferences in complex systems. Section 4.1 shows how risk sensitivity emerges in our model, from both genetic and cultural adaptation. It confirms findings from pre-existent literature [26, 8, 9]. What is more, it shows how both risk aversion and risk-seeking can emerge from the adaptation process of a risk-neutral population of simple agents.

5.1 Relationships between the emergence of risk sensitivity and environmental features

Section 4.2 deepens the relationships between the emergence of risk sensitivity and environmental features. Adaptation styles can alter these relationships in a non-trivial way. Environmental parameters can have different effects when agents adjust to the environment through learning or through genetic evolution. Table 3 offers a summary of this effect and highlights three cases that are discussed in what follows.

Table 3. parameters clustering for value of d in different scenario

	$LS < 0$	$LS > 0$
$ES < 0$	$w\_size, rr, e\_num, e\_lev, e\_cons$	v, l_avg, e_max
$ES > 0$	e_in, mm, att_l	a_max, att_c, att_g

ES < 0 and LS < 0. Parameters in this group have a negative influence on the resulted risk sensitivity independently from the adaptation style. All the parameters related to the energy level of the system are in this category. It means that the lower is the energy level, the higher is the probability for a population to become risk-seeking. The world size is also in this category. A possible explanation is that it influences the chance that an agent reaches an energy source.

ES > 0 and LS > 0. Parameters in this cluster always have a positive influence on risk sensitivity. It means that the lower they are, the higher is the chance that agents become risk-prone, no matter how they adapt. Parameters in this group are related to the expected results of a risky decision. A potential interpretation is that the adaptation process generates risk-seeking agents when the expected result of a risky decision is higher. Within this set, there is the maximum number of agents in a simulation. Hintze et al. 2015 suggest that, under evolutionary pressure, the number of interacting agents strongly influence the emergence of risk aversion [14]. Our results confirm that finding. Furthermore, we propose that they are valid for both genetic and cultural adaptation.

Mixed. In this group, the adaptation technique influences the effect of the parameters on the risk sensitivity. In some cases, it is due to the model design. For example, the length of the memory is not significant in the ES scenario. It is expected since memory is not involved in the evolutionary process. Other parameters, such as the vision radius and the average life duration of agents, have a non-trivial explanation. The vision radius has a negative influence in the ES scenario and a positive on in the LS. A possible explanation is connected to the learning process of agents. Lower vision means agents take less decision during their existences. Thus, under cultural pressure agents have less chance to determine how to deal with risk. Hence, the population become biased (less risk-seeking than necessary). Under genetic pressure, the decision process is not dynamic, and the only issue is to collect energy. A lower vision ray implies less chance to get it. Consequently, the wider is the vision ray, the more likely is for risk-averse agents to survive and generate offsprings.

We interpret the results regarding the average duration of the agent's life using the notion of the pace of life. A shorter life means a lower expected number of offsprings to lose in case an attack fails. Under genetic pressure, this drives agents toward a less conservative mindset. In case agents learn, longer lives allow them to evaluate better the expected outcome of a solution. It pushes the population towards a less biased set of risk preferences (in this case, more risk-prone). These results address a need for a wider perspective on evolutionary approaches to individuals' decision-making that also includes cultural adaptation in the process of shaping risk preferences [3].

5.2 Relationship between the emergence of risk sensitivity and the harshness of the environment

Section 4.3 displays how the harshness of the environment influences risk preferences alteration. For both cultural and genetic adaptation, there is a non-monotonic relationship between these two factors. Szpiro founds something similar [26]. It analysed an elementary stock market in which agents genetically adapt and compensate for the increase of environmental danger with a higher number of offsprings per time step. Results from this model are comparable when adaptation is driven mostly by genetic selection. In that case, they both show a direct relationship between the harshness of the environment and the final level of risk aversion. Also, both point that this phenomenon is less intense when the rate of survival is low. Here we propose a possible explanation with two concurring forces. The first is fitness for the environment. When there is no risk, entities have no incentives to develop risk preferences. This incentive grows with the dangerousness of the surroundings. The second is the expected benefit of risk-aware behaviour. Being more cautious is progressively less beneficial when the environment is harsher. There could be a level of dangerousness of the environment in which the surroundings are so perilous that being risk-averse does not change the chance of avoiding risk. We suggest that the non-linear and non-monotonic behaviour of this relationship is the combination of these two strengths. While having these properties, the shape of the curve can vary. This difference could be addressed to various combination patterns existing in distinct systems. Further analysis is necessary to prove these hypotheses.

6 Conclusions

Our work improves upon the existing knowledge about the emergence of risk preferences in complex systems. It shows that nontrivial relationships can occur between environmental factors and the emergence of risk sensitivity in MABS. What is more, it explains how the style of adaptation influences these relationships. Since the simulated model is simple, we claim these results could apply to various application fields.

The research has limitations and several potential extensions. The environmental parameters are static. This is partially compensated by the co-evolution dynamics of the agents. Further investigations could acknowledge it. Similarly, we consider only a single learning strategy, a simple one. It is coherent with the goal of the research. Future developments could assess the effect of different learning approaches. To conclude, potential outlooks include the generalization of the results, the identification of a connection between the pace of life and the emergence of risk preferences, and a spatial network representation of the problem [1].

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